Package 'BiDAG'

May 16, 2023

Type Package

Title Bayesian Inference for Directed Acyclic Graphs

Version 2.1.4

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Description Implementation of a collection of MCMC methods for Bayesian structure learning of directed acyclic graphs (DAGs), both from continuous and discrete data. For efficient inference on larger DAGs, the space of DAGs is pruned according to the data. To filter the search space, the algorithm employs a hybrid approach, combining constraint-based learning with search and score. A reduced search space is initially defined on the basis of a skeleton obtained by means of the PC-algorithm, and then iteratively improved with search and score. Search and score is then performed following two approaches: Order MCMC, or Partition MCMC.

The BGe score is implemented for continuous data and the BDe score is implemented for binary data or categorical data. The algorithms may provide the maximum a posteriori (MAP) graph or a sample (a collection of DAGs) from the posterior distribution given the data. All algorithms are also applicable for structure learning and sampling for dynamic Bayesian networks.

References:

J. Kuipers, P. Suter, G. Moffa (2022) <doi:10.1080/10618600.2021.2020127>,

N. Friedman and D. Koller (2003) <doi:10.1023/A:1020249912095>,

J. Kuipers and G. Moffa (2017) <doi:10.1080/01621459.2015.1133426>,

M. Kalisch et al. (2012) <doi:10.18637/jss.v047.i11>,

D. Geiger and D. Heckerman (2002) <doi:10.1214/aos/1035844981>,

P. Suter, J. Kuipers, G. Moffa, N.Beerenwinkel (2023) <doi:10.18637/jss.v105.i09>.

Acknowledgments We would like to thank Giusi Moffa for discussion and comments on the package and its manual.

License GPL (>= 2)

Depends R (>= 3.5.0)

Imports Rcpp (>= 0.12.7), methods, graph, Rgraphviz, RBGL, pcalg, graphics, Matrix, coda

LinkingTo Rcpp **RoxygenNote** 7.2.0 Encoding UTF-8
LazyData TRUE
NeedsCompilation yes
Repository CRAN
Date/Publication 2023-05-16 12:46:02 UTC

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Description

A synthetic dataset from Lauritzen and Spiegelhalter (1988) about lung diseases (tuberculosis, lung cancer or bronchitis) and visits to Asia.

Usage

Asia

Format

A data frame with 5000 rows and 8 binary variables:

- D (dyspnoea), binary 1/0 corresponding to "yes" and "no"
- T (tuberculosis), binary 1/0 corresponding to "yes" and "no"
- L (lung cancer), binary 1/0 corresponding to "yes" and "no"
- B (bronchitis), binary 1/0 corresponding to "yes" and "no"
- A (visit to Asia), binary 1/0 corresponding to "yes" and "no"
- S (smoking), binary 1/0 corresponding to "yes" and "no"
- X (chest X-ray), binary 1/0 corresponding to "yes" and "no"
- E (tuberculosis versus lung cancer/bronchitis), binary 1/0 corresponding to "yes" and "no"

Source

https://www.bnlearn.com/bnrepository/

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References

Lauritzen S, Spiegelhalter D (1988). 'Local Computation with Probabilities on Graphical Structures and their Application to Expert Systems (with discussion)'. Journal of the Royal Statistical Society: Series B 50, 157-224.

Asiamat

Asiamat

Description

An adjacency matrix representing the ground truth DAG used to generate a synthetic dataset from Lauritzen and Spiegelhalter (1988) about lung diseases (tuberculosis, lung cancer or bronchitis) and visits to Asia.

Usage

Asiamat

Format

A binary matrix with 8 rows and 8 columns representing an adjacency matrix of a DAG with 8 nodes:

- D (dyspnoea), binary 1/0 corresponding to "yes" and "no"
- T (tuberculosis), binary 1/0 corresponding to "yes" and "no"
- L (lung cancer), binary 1/0 corresponding to "yes" and "no"
- B (bronchitis), binary 1/0 corresponding to "yes" and "no"
- A (visit to Asia), binary 1/0 corresponding to "yes" and "no"
- S (smoking), binary 1/0 corresponding to "yes" and "no"
- X (chest X-ray), binary 1/0 corresponding to "yes" and "no"
- E (tuberculosis versus lung cancer/bronchitis), binary 1/0 corresponding to "yes" and "no"

Source

https://www.bnlearn.com/bnrepository/

References

Lauritzen S, Spiegelhalter D (1988). 'Local Computation with Probabilities on Graphical Structures and their Application to Expert Systems (with discussion)'. Journal of the Royal Statistical Society: Series B 50, 157-224.

bidag2coda 5

bidag2coda	Converting a single BiDAG chain to mcmc object	

Description

This function converts a single object of one of the BiDAG classes, namely 'orderMCMC' or 'partitionMCMC' to an object of class 'mcmc'. This object can be further used for convergence and mixing diagnostics implemented in the package coda

Usage

```
bidag2coda(
  MCMCtrace,
  edges = FALSE,
  pdag = TRUE,
  p = 0.1,
  burnin = 0.2,
  window = 100,
  cumulative = FALSE
)
```

Arguments

MCMCtrace	object of class orderMCMC or partitionMCMC
edges	logical, when FALSE (default), then only DAG score trace is extracted; when TRUE, a trace of posterior probabilities is extracted for every edge (based on the sampled DAGs defined by parameters 'window' and 'cumulative') resulting in up to n^2 trace vectors, where n is the number of nodes in the network
pdag	logical, when edges=TRUE, defines if the DAGs are converted to CPDAGs prior to computing posterior probabilities; ignored otherwise
p	numeric, between 0 and 1; defines the minimum probability for including posterior traces in the returned objects (for probabilities close to 0 PRSF diagnostics maybe too conservative)
burnin	numeric between \emptyset and 1, indicates the percentage of the samples which will be discarded as 'burn-in' of the MCMC chain; the rest of the samples will be used to calculate the posterior probabilities; 0.2 by default
window	integer, defines a number of DAG samples for averaging and computing edges' posterior probabilities; ignored when edges=FALSE
cumulative	logical, indicates if posterior probabilities should be calculated based on a cumulative sample of DAGs, where 25% of the first samples are discarded

Value

Object of class mcmc from the package coda

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Author(s)

Polina Suter

Examples

```
## Not run:
library(coda)
myscore<-scoreparameters("bde",Asia)
ordersample<-sampleBN(myscore, "order")
order_mcmc<-bidag2coda(ordersample)
par(mfrow=c(1,2))
densplot(order_mcmc)
traceplot(order_mcmc)</pre>
## End(Not run)
```

bidag2codalist

Converting multiple BiDAG chains to mcmc.list

Description

This function converts a list of objects of classes 'orderMCMC' or 'partitionMCMC' to an object of class 'mcmc.list'. This object can be further used for convergence and mixing diagnostics implemented in the R-package coda.

Usage

```
bidag2codalist(
  MCMClist,
  edges = FALSE,
  pdag = TRUE,
  p = 0.1,
  burnin = 0.2,
  window = 10,
  cumulative = FALSE
)
```

Arguments

MCMClist a list of objects of classes orderMCMC or partitionMCMC

edges logical, when FALSE (default), then only DAG score trace is extracted; when

TRUE, a trace of posterior probabilities is extracted for every edge (based on the sampled DAGs defined by parameters 'window' and 'cumulative') resulting in

up to n^2 trace vectors, where n is the number of nodes in the network

pdag logical, when edges=TRUE, defines if the DAGs are converted to CPDAGs prior

to computing posterior probabilities; ignored otherwise

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p	numeric, between 0 and 1; defines the minimum probability for including posterior traces in the returned objects (for probabilities close to 0, PRSF diagnostics maybe too conservative; the threshold above 0 is recommended)
burnin	numeric between 0 and 1, indicates the percentage of the samples which will be discarded as 'burn-in' of the MCMC chain; the rest of the samples will be used to calculate the posterior probabilities; 0.2 by default
window	integer, defines a number of DAG samples for averaging and computing edges' posterior probabilities; ignored when edges=FALSE
cumulative	logical, indicates if posterior probabilities should be calculated based on a cumulative sample of DAGs, where 25% of the first samples are discarded

Value

Object of class mcmc.list from the package coda

Author(s)

Polina Suter

References

Robert J. B. Goudie and Sach Mukherjee (2016). A Gibbs Sampler for Learning DAGs. J Mach Learn Res. 2016 Apr; 17(30): 1–39.

Examples

```
## Not run:
library(coda)
scoreBoston<-scoreparameters("bge",Boston)
ordershort<-list()
#run very short chains -> convergence issues
ordershort[[1]] <- sampleBN(scoreBoston, algorithm = "order", iterations=2000)
ordershort[[2]] <- sampleBN(scoreBoston, algorithm = "order", iterations=2000)
codashort_edges<-bidag2codalist(ordershort,edges=TRUE,pdag=TRUE,p=0.05,burnin=0.2,window=10)
gd_short<-gelman.diag(codashort_edges, transform=FALSE, autoburnin=FALSE, multivariate=FALSE)
length(which(gd_short$psrf[,1]>1.1))/(length(gd_short$psrf[,1]))
#=>more MCMC iterations are needed, try 100000
```

Boston	Boston housing data

Description

A dataset containing information collected by the U.S Census Service concerning housing in the area of Boston, originally published by Harrison and Rubinfeld (1978).

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Usage

Boston

Format

A data frame with 506 rows and 14 variables:

- CRIM per capita crime rate by town
- ZN proportion of residential land zoned for lots over 25,000 sq.ft.
- INDUS proportion of non-retail business acres per town.
- CHAS Charles River dummy variable (1 if tract bounds river; 0 otherwise)
- NOX nitric oxides concentration (parts per 10 million)
- RM average number of rooms per dwelling
- AGE proportion of owner-occupied units built prior to 1940
- DIS weighted distances to five Boston employment centres
- TAX full-value property-tax rate per \$10,000
- RAD index of accessibility to radial highways
- PTRATIO pupil-teacher ratio by town
- B 1000(Bk 0.63)² where Bk is the proportion of blacks by town
- LSTAT percentage lower status of the population
- MEDV Median value of owner-occupied homes in \$1000's

Source

http://lib.stat.cmu.edu/datasets/boston

References

Harrison, D and Rubinfeld, DL (1978) 'Hedonic prices and the demand for clean air', Journal of Environmental Economics and Management 5, 81-102.

compact2full

Deriving an adjecency matrix of a full DBN

Description

This function transforms a compact 2-slice adjacency matrix of DBN into full T-slice adjacency matrix

Usage

```
compact2full(DBNmat, slices, b = 0)
```

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Arguments

DBNmat	a square matrix, representing initial and transitional structure of a DBN; the size
	of matrix is 2*dyn+b

slices integer, number of slices in an unrolled DBN

b integer, number of static variables

Value

an adjacency matrix of an unrolled DBN

Examples

```
compact2full(DBNmat, slices=5, b=3)
```

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Comparing two graphs

Description

This function compares one (estimated) graph to another graph (true graph), returning a vector of 8 values:

- the number of true positive edges ('TP') is the number of edges in the skeleton of 'egraph' which are also present in the skeleton of 'truegraph'
- the number of false positive edges ('FP') is the number of edges in the skeleton of 'egraph' which are absent in the skeleton of 'truegraph'
- the number of fralse negative edges ('FN') is the number of edges in the skeleton of 'truegraph' which are absent in the skeleton of 'egraph'
- structural Hamming distance ('SHD') between 2 graphs is computed as TP+FP+the number of edges with an error in direction
- TPR equals TP/(TP+FN)
- FPR equals FP/(TN+FP) (TN stands for true negative edges)
- FPRn equals FP/(TP+FN)
- FDR equals FP/(TP+FP)

Usage

```
compareDAGs(egraph, truegraph, cpdag = FALSE, rnd = 2)
```

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Arguments

egraph an object of class graphNEL (package 'graph'), representing the graph which

should be compared to a ground truth graph or an ajecency matrix corresponding

to the graph

truegraph an object of class graphNEL (package 'graph'), representing the ground truth

graph or an ajecency matrix corresponding to this graph

cpdag logical, if TRUE (FALSE by default) both graphs are first converted to their

respective equivalence class (CPDAG); this affects SHD calculation

rnd integer, rounding integer indicating the number of decimal places (round) when

computing TPR, FPR, FPRn and FDR

Value

a named numeric vector 8 elements: SHD, number of true positive edges (TP), number of false positive edges (FP), number of false negative edges (FN), true positive rate (TPR), false positive rate (FPR), false positive rate normalized to the true number of edges (FPRn) and false discovery rate (FDR)

Examples

```
Asiascore<-scoreparameters("bde", Asia)
## Not run:
eDAG<-learnBN(Asiascore,algorithm="order")
compareDAGs(eDAG$DAG,Asiamat)
## End(Not run)
```

compareDBNs

Comparing two DBNs

Description

This function compares one (estimated) DBN structure to another DBN (true DBN). Comparisons for initial and transitional structures are returned separately if equalstruct equals TRUE.

Usage

```
compareDBNs(eDBN, trueDBN, struct = c("init", "trans"), b = 0)
```

Arguments

eDBN an object of class graphNEL (or an ajacency matrix corresponding to this DBN),

representing the DBN which should be compared to a ground truth DBN

trueDBN an object of class graphNEL (or an ajacency matrix corresponding to this DBN),

representing the ground truth DBN

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struct	option used to determine if the initial or the transitional structure should be compared; accaptable values are init or trans
b	number of static variables in one time slice of a DBN; note that for function to work correctly all static variables have to be in the first b columns of the matrix

Value

a vector of 5: SHD, number of true positive edges, number of false positive edges, number of false negative edges and true positive rate

Examples

```
testscore<-scoreparameters("bge", DBNdata, DBN=TRUE,
dbnpar=list(samestruct=TRUE, slices=5, b=3))
## Not run:
DBNfit<-learnBN(testscore, algorithm="orderIter",moveprobs=c(0.11,0.84,0.04,0.01))
compareDBNs(DBNfit$DAG,DBNmat, struct="trans", b=3)
## End(Not run)</pre>
```

connectedSubGraph

Deriving connected subgraph

Description

This function derives an adjacency matrix of a subgraph whose nodes are connected to at least one other node in a graph

Usage

```
connectedSubGraph(adj)
```

Arguments

adj

square adjacency matrix with elements in {0,1}, representing a graph

Value

adjacency matrix of a subgraph of graph represented by 'adj' whose nodes have at least one connection

Examples

```
dim(gsimmat) #full graph contains 100 nodes
gconn<-connectedSubGraph(gsimmat) #removing disconnected nodes
dim(gconn) #connected subgraph contains 93 nodes</pre>
```

DAGscore

DAGscore	Calculating the BGe/BDe score of a single DAG

Description

This function calculates the score of a DAG defined by its adjacency matrix. Acceptable data matrices are homogeneous with all variables of the same type: continuous, binary or categorical. The BGe score is evaluated in the case of continuous data and the BDe score is evaluated for binary and categorical variables.

Usage

```
DAGscore(scorepar, incidence)
```

Arguments

scorepar an object of class scoreparameters, containing the data and scoring parame-

ters; see constructor function scoreparameters

incidence a square matrix of dimensions equal to the number of nodes, representing the ad-

jacency matrix of a DAG; the matrix entries are in $\{0,1\}$ such that incidence [i,j] equals 1 if there is a directed edge from node i to node j in the DAG and

incidence[i,j] equals 0 otherwise

Value

the log of the BGe or BDe score of the DAG

Author(s)

Jack Kuipers, Polina Suter, the code partly derived from the order MCMC implementation from Kuipers J, Moffa G (2017) <doi:10.1080/01621459.2015.1133426>

References

Geiger D and Heckerman D (2002). Parameter priors for directed acyclic graphical models and the characterization of several probability distributions. The Annals of Statistics 30, 1412-1440.

Heckerman D and Geiger D (1995). Learning Bayesian networks: A unification for discrete and Gaussian domains. In Eleventh Conference on Uncertainty in Artificial Intelligence, pages 274-284.

Kuipers J, Moffa G and Heckerman D (2014). Addendum on the scoring of Gaussian directed acyclic graphical models. The Annals of Statistics 42, 1689-1691.

Examples

```
myScore<-scoreparameters("bde", Asia)
DAGscore(myScore, Asiamat)</pre>
```

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DBNdata

Simulated data set from a 2-step dynamic Bayesian network

Description

A synthetic dataset containing 100 observations generated from a random dynamic Bayesian network with 12 continuous dynamic nodes and 3 static nodes. The DBN includes observations from 5 time slices.

Usage

DBNdata

Format

A data frame with 100 rows and 63 (3+12*5) columns representing observations of 15 variables: 3 static variables (first 3 columns) which do not change over time and 12 dynamic variables observed in 5 consecutive time slices.

DBNmat

An adjacency matrix of a dynamic Bayesian network

Description

An adjacency matrix representing the ground truth DBN used to generate a synthetic dataset DBNdata. The matrix is a compact representation of a 2-step DBN, such that initial structure is stored in the first 15 columns of the matrix and transitional structure is stored in the last 12 columns of the matrix.

Usage

DBNmat

Format

A binary matrix with 27 rows and 27 columns representing an adjacency matrix of a DBN. Rows and columns of the matrix correspond to 15 variables of a DBN across 2 time slices.

DBNscore

DBNscore

Calculating the BGe/BDe score of a single DBN

Description

This function calculates the score of a DBN defined by its compact adjacency matrix. Acceptable data matrices are homogeneous with all variables of the same type: continuous, binary or categorical. The BGe score is evaluated in the case of continuous data and the BDe score is evaluated for binary and categorical variables.

Usage

DBNscore(scorepar, incidence)

Arguments

scorepar an object of class scoreparameters, containing the data and scoring parame-

ters; see constructor function scoreparameters

incidence a square matrix, representing initial and transitional structure of a DBN; the

size of matrix is 2*nsmall+bgn, where nsmall is the number of variables per time slice excluding static nodes and bgn is the number of static variables the matrix entries are in {0,1} such that incidence[i,j] equals 1 if there is a directed edge from node i to node j in the DAG and incidence[i,j] equals 0

otherwise

Value

the log of the BGe or BDe score of the DBN

Author(s)

Polina Suter, Jack Kuipers

Examples

```
testscore<-scoreparameters("bge", DBNdata, DBN=TRUE, dbnpar=list(slices=5, b=3))
DBNscore(testscore, DBNmat)</pre>
```

DBNunrolled 15

DBNunrolled	An unrolled adjacency matrix of a dynamic Bayesian network

Description

An adjacency matrix representing the ground truth DBN used to generate a synthetic dataset DBNdata. The matrix is an unrolled representation of a 2-step DBN, such that the static variables are represented in the first 3 columns/rows of the matrix.

Usage

DBNunrolled

Format

A binary matrix with 63 rows and 63 columns representing an adjacency matrix of a DBN. Rows and columns of the matrix correspond to 15 variables (s1, s2, s3, v1, v2, v3, v4, v5, v6, v7, v8, v9, v10, v11, v12) of a DBN across 5 time slices.

edgep	Estimating posterior probabilities of single edges
3.0.1	J

Description

This function estimates the posterior probabilities of edges by averaging over a sample of DAGs obtained via an MCMC scheme.

Usage

```
edgep(MCMCchain, pdag = FALSE, burnin = 0.2, endstep = 1)
```

Arguments

MCMCchain	an object of class partitionMCMC, orderMCMC or iterativeMCMC, representing the output of structure sampling function partitionMCMC or orderMCMC (the latter when parameter chainout=TRUE;
pdag	logical, if TRUE (FALSE by default) all DAGs in the MCMCchain are first converted to equivalence class (CPDAG) before the averaging
burnin	number between 0 and 1, indicates the percentage of the samples which will be discarded as 'burn-in' of the MCMC chain; the rest of the samples will be used to calculate the posterior probabilities; 0.2 by default
endstep	number between 0 and 1; 1 by default

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Value

a square matrix with dimensions equal to the number of variables; each entry [i,j] is an estimate of the posterior probability of the edge from node i to node j

Author(s)

Polina Suter

Examples

```
Bostonscore<-scoreparameters("bge", Boston)
## Not run:
samplefit<-sampleBN(Bostonscore, "order")
edgesposterior<-edgep(samplefit, pdag=TRUE, burnin=0.2)
## End(Not run)</pre>
```

full2compact

Deriving a compact adjacency matrix of a DBN

Description

This function transforms an unrolled adjacency matrix of DBN into a compact representation

Usage

```
full2compact(DBNmat, b = 0)
```

Arguments

DBNmat a square matrix, representing the structure of an unrolled DBN; the size of matrix

is slices*dyn+b; all static variables are assumed to be in the first b rows and

columns of the matrix

b integer, number of static variables; 0 by default

Examples

```
full2compact(DBNunrolled, b=3)
```

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getDAG

Extracting adjacency matrix (DAG) from MCMC object

Description

This function extracts an adjacency matrix of a maximum scoring DAG from the result of the MCMC run.

Usage

```
getDAG(x, amat = TRUE, cp = FALSE)
```

Arguments

x object of class 'orderMCMC', 'partitionMCMC' or 'iterativeMCMC'

amat logical, when TRUE adjacency matrix is returned and object of class 'graph-

NEL' otherwise

cp logical, when TRUE the CPDAG (equivalence class) is returned and DAG oth-

erwise; FALSE by default

Value

adjacency matrix of a maximum scoring DAG (or CPDAG) discovered/sampled in one MCMC run

Examples

```
myscore<-scoreparameters("bge", Boston)
## Not run:
itfit<-learnBN(myscore,algorithm="orderIter")
maxEC<-getDAG(itfit,cp=TRUE)
## End(Not run)</pre>
```

getMCMCscore

Extracting score from MCMC object

Description

This function extracts the score of a maximum DAG sampled in the MCMC run.

Usage

```
getMCMCscore(x)
```

Arguments

x object of class 'orderMCMC', 'partitionMCMC' or 'iterativeMCMC'

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Value

a score of a maximum-scoring DAG found/sampled in one MCMC run

Examples

```
myscore<-scoreparameters("bge", Boston)
## Not run:
itfit<-learnBN(myscore,algorithm="orderIter")
getMCMCscore(itfit)
## End(Not run)</pre>
```

getRuntime

Extracting runtime

Description

This function extracts runtime of a particular step of order and partition MCMC.

Usage

```
getRuntime(x, which = 0)
```

Arguments

x object of class 'orderMCMC' or 'partitionMCMC'

which integer, defines if the runtime is extracted for: computing score tables (which =

1), running MCMC chain (which = 2)

Value

runtime of a particular step of MCMC scheme or total runtime

Examples

```
myscore<-scoreparameters("bge",Boston)
## Not run:
orderfit<-sampleBN(myscore,algorithm="order")
(getRuntime(orderfit,1))
(getRuntime(orderfit,2))
## End(Not run)</pre>
```

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getSpace

Extracting scorespace from MCMC object

Description

This function extracts an object of class 'scorespace' from the result of the MCMC run when the parameter 'scoreout' was set to TRUE; otherwise extracts only adjacency matrix of the final search space without the score tables.

Usage

```
getSpace(x)
```

Arguments

Х

object of class 'orderMCMC', 'partitionMCMC' or 'iterativeMCMC'

Value

an object of class 'scorespace' or an adjacency binary matrix corresponding to a search space last used in MCMC

Examples

```
myscore<-scoreparameters("bge", Boston)
## Not run:
itfit<-learnBN(myscore,algorithm="orderIter",scoreout=TRUE)
itspace<-getSpace(itfit)
## End(Not run)</pre>
```

getSubGraph

Deriving subgraph

Description

This function derives an adjacency matrix of a subgraph based on the adjacency matrix of a full graph and a list of nodes

Usage

```
getSubGraph(adj, nodes)
```

Arguments

adj square adjacency matrix with elements in {0,1}, representing a graph

nodes vector of node names of the subgraph; should be a subset of column names of

'adj'

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Value

adjacency matrix of a subgraph which includes all 'nodes'

Examples

```
getSubGraph(Asiamat,c("E","B","D","X"))
```

getTrace

Extracting trace from MCMC object

Description

This function extracts a trace of

- DAG scores
- · DAG adjacency matrices
- · orders
- · order scores

from the result of the MCMC run. Note that the last three options work only when the parameter 'scoreout' was set to TRUE.

Usage

```
getTrace(x, which = 0)
```

Arguments

x object of class 'orderMCMC', 'partitionMCMC' or 'iterativeMCMC'
which integer, indication which trace is returned: DAG scores (which = 0), DAGs
(which = 1), orders (which = 2), order scores (which = 3)

Value

a list or a vector of objects representing MCMC trace, depends on parameter 'which'; by default, the trace of DAG scores is returned

Examples

```
myscore<-scoreparameters("bge",Boston)
## Not run:
orderfit<-sampleBN(myscore,algorithm="order")
DAGscores<-getTrace(orderfit,which=0)
DAGtrace<-getTrace(orderfit,which=1)
orderscores<-getTrace(orderfit,which=3)
## End(Not run)</pre>
```

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graph2m

Deriving an adjacency matrix of a graph

Description

This function derives the adjacency matrix corresponding to a graph object

Usage

```
graph2m(g)
```

Arguments

g

graph, object of class graphNEL (package 'graph')

Value

a square matrix whose dimensions are the number of nodes in the graph g, where element [i,j] equals 1 if there is a directed edge from node i to node j in the graph g, and 0 otherwise

Examples

```
Asiagraph<-m2graph(Asiamat)
Asia.adj<-graph2m(Asiagraph)
```

gsim

A simulated data set from a Gaussian continuous Bayesian network

Description

A synthetic dataset containing 1000 observations generated from a random DAG with 100 continuous nodes. Functions 'randomDAG' and 'rmvDAG' from R-packages 'pcalg' were used to generate the data.

Usage

gsim

Format

A data frame with 1000 rows representing observations of 100 continuous variables: V1, ..., V100

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gsim100

A simulated data set from a Gaussian continuous Bayesian network

Description

A synthetic dataset containing 100 observations generated from a random DAG with 100 continuous nodes. Functions 'randomDAG' and 'rmvDAG' from R-packages 'pcalg' were used to generate the data.

Usage

gsim100

Format

A data frame with 100 rows representing observations of 100 continuous variables: V1, ..., V100

gsimmat

An adjacency matrix of a simulated dataset

Description

An adjacency matrix representing the ground truth DAG used to generate a synthetic dataset with observations of 100 continuous variables.

Usage

gsimmat

Format

A binary matrix with 100 rows and 100 columns representing an adjacency matrix of a DAG with 100 nodes: V1, ..., V100

interactions 23

interactions	inter
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interactions dataset

Description

A data frame containing possible interactions between genes from kirp and kirc data sets

Usage

interactions

Format

A data frame with 179 rows and 3 columns;

- node1 character, name of a gene
- node2 character, name of a gene
- combined_score interaction score, reflecting confidence in the fact that interaction between gene1 and gene2 is possible

each row represents a possible interaction between two genes

Source

```
https://string-db.org/
```

iterativeMCMC	Structure learning with an iterative order MCMC algorithm on an ex-
	panded search space

Description

This function implements an iterative search for the maximum a posteriori (MAP) DAG, by means of order MCMC (arXiv:1803.07859v3). At each iteration, the current search space is expanded by allowing each node to have up to one additional parent not already included in the search space. By default the initial search space is obtained through the PC-algorithm (using the functions skeleton and pc from the 'pcalg' package [Kalisch et al, 2012]). At each iteration order MCMC is employed to search for the MAP DAG. The edges in the MAP DAG are added to the initial search space to provide the search space for the next iteration. The algorithm iterates until no further score improvements can be achieved by expanding the search space. The final search space may be used for the sampling versions of orderMCMC and partitionMCMC.

Usage

```
iterativeMCMC(
  scorepar,
 MAP = TRUE,
  posterior = 0.5,
  softlimit = 9,
  hardlimit = 12,
  alpha = 0.05,
  gamma = 1,
  verbose = TRUE,
  chainout = FALSE,
  scoreout = FALSE,
  cpdag = FALSE,
 mergetype = "skeleton",
  iterations = NULL,
 moveprobs = NULL,
  stepsave = NULL,
  startorder = NULL,
  accum = FALSE,
  compress = TRUE,
  plus1it = NULL,
  startspace = NULL,
  blacklist = NULL,
  addspace = NULL,
  scoretable = NULL,
  alphainit = NULL
)
## S3 method for class 'iterativeMCMC'
plot(
 х,
 main = "iterative MCMC, DAG scores",
 xlab = "MCMC step",
 ylab = "DAG logscore",
  type = "1",
  col = "blue"
)
## S3 method for class 'iterativeMCMC'
print(x, ...)
## S3 method for class 'iterativeMCMC'
summary(object, ...)
```

Arguments

scorepar an object of class scoreparameters, containing the data and scoring parame-

ters; see constructor function scoreparameters

MAP logical, if TRUE (default) the search targets the MAP DAG (a DAG with max-

imum score), if FALSE at each MCMC step a DAG is sampled from the order proportionally to its score; when expanding a search space when MAP=TRUE all edges from the maximum scoring DAG are added to the new space, when MAP=FALSE only edges with posterior probability higher than defined by pa-

rameter posterior are added to the search space

posterior logical, when MAP set to FALSE defines posterior probability threshold for adding

the edges to the search space

softlimit integer, limit on the size of parent sets beyond which adding undirected edges is

restricted; below this limit edges are added to expand the parent sets based on the undirected skeleton of the MAP DAG (or from its CPDAG, depending on the parameter mergecp), above the limit only the directed edges are added from

the MAP DAG; the limit is 9 by default

hardlimit integer, limit on the size of parent sets beyond which the search space is not

further expanded to prevent long runtimes; the limit is 12 by default

alpha numerical significance value in $\{0,1\}$ for the conditional independence tests in

the PC-stage

gamma tuning parameter which transforms the score by raising it to this power, 1 by

default

verbose logical, if TRUE (default) prints messages on the progress of execution

chainout logical, if TRUE the saved MCMC steps are returned, FALSE by default

scoreout logical, if TRUE the search space from the last plus1 iterations and the corre-

sponding score tables are returned, FALSE by default

cpdag logical, if set to TRUE the equivalence class (CPDAG) found by the PC algo-

rithm is used as a search space, when FALSE (default) the undirected skeleton

used as a search space

mergetype defines which edges are added to the search space at each expansion iteration;

three options are available 'dag', 'cpdag', 'skeleton'; 'skeleton' by default

iterations integer, the number of MCMC steps, the default value is $3.5n^2 \log n$

moveprobs a numerical vector of 4 values in $\{0,1\}$ corresponding to the probabilities of the

following MCMC moves in the order space:

• exchanging 2 random nodes in the order

• exchanging 2 adjacent nodes in the order

• placing a single node elsewhere in the order

• staying still

stepsave integer, thinning interval for the MCMC chain, indicating the number of steps

between two output iterations, the default is iterations/1000

startorder integer vector of length n, which will be used as the starting order in the MCMC

algorithm, the default order is random

logical, when TRUE at each search step expansion new edges are added to the accum current search space; when FALSE (default) the new edges are added to the

starting space

logical, if TRUE adjacency matrices representing sampled graphs will be stored compress

as a sparse Matrix (recommended); TRUE by default

plus1it (optional) integer, a number of iterations of search space expansion; by default

the algorithm iterates until no score improvement can be achieved by further

expanding the search space

(optional) a square matrix, of dimensions equal to the number of nodes, which startspace

> defines the search space for the order MCMC in the form of an adjacency matrix; if NULL, the skeleton obtained from the PC-algorithm will be used; if startspace[i,j] equals to 1 (0) it means that the edge from node i to node j is included (excluded) from the search space; to include an edge in both direc-

tions, both startspace[i,j] and startspace[j,i] should be 1

(optional) a square matrix, of dimensions equal to the number of nodes, which defines edges to exclude from the search space; if blacklist[i,j] equals to 1 it means that the edge from node i to node j is excluded from the search space

• "dag", then edges from maximum scoring DAG are added;

• "cpdag", then the maximum scoring DAG is first converted to the CPDAG, from which all edges are added to the search space;

• "skeleton", then the maximum scoring DAG is first converted to the skeleton, from which all edges are added to the search space

addspace (optional) a square matrix, of dimensions equal to the number of nodes, which

> defines the edges, which are added at to the search space only at the first iteration of iterative seach and do not necessarily stay afterwards; defined in the form of an adjacency matrix; if addspace[i,j] equals to 1 (0) it means that the edge from node i to node j is included (excluded) from the search space; to include an edge in both directions, both addspace[i,j] and addspace[j,i] should be

scoretable (optional) object of class scorespace. When not NULL, parameters startspace

and addspace are ignored.

(optional) numerical, defines alpha that is used by the PC algorithm to learn alphainit

initial structure of a DBN, ignored in static case

object of class 'iterativeMCMC' Х

ignored

name of the graph; "iterative MCMC, DAG scores" by default main

name of x-axis; "MCMC step" xlab

name of y-axis; "DAG logscore" ylab

type of line in the plot; "l" by default type

col colour of line in the plot; "blue" by default

object object of class 'iterativeMCMC'

blacklist

Value

Object of class iterativeMCMC, which contains log-score trace as well as adjacency matrix of the maximum scoring DAG, its score and the order score. The output can optionally include DAGs sampled in MCMC iterations and the score tables. Optional output is regulated by the parameters chainout and scoreout. See iterativeMCMC class for a detailed class structure.

Note

see also extractor functions getDAG, getTrace, getSpace, getMCMCscore.

Author(s)

Polina Suter, Jack Kuipers

References

P. Suter, J. Kuipers, G. Moffa, N.Beerenwinkel (2023) <doi:10.18637/jss.v105.i09>

Kuipers J, Super P and Moffa G (2020). Efficient Sampling and Structure Learning of Bayesian Networks. (arXiv:1803.07859v3)

Friedman N and Koller D (2003). A Bayesian approach to structure discovery in bayesian networks. Machine Learning 50, 95-125.

Kalisch M, Maechler M, Colombo D, Maathuis M and Buehlmann P (2012). Causal inference using graphical models with the R package pealg. Journal of Statistical Software 47, 1-26.

Geiger D and Heckerman D (2002). Parameter priors for directed acyclic graphical models and the characterization of several probability distributions. The Annals of Statistics 30, 1412-1440.

Kuipers J, Moffa G and Heckerman D (2014). Addendum on the scoring of Gaussian directed acyclic graphical models. The Annals of Statistics 42, 1689-1691.

Spirtes P, Glymour C and Scheines R (2000). Causation, Prediction, and Search, 2nd edition. The MIT Press.

Examples

```
## Not run:
Bostonpar<-scoreparameters("bge",Boston)
itfit<-iterativeMCMC(Bostonpar, chainout=TRUE, scoreout=TRUE)
plot(itfit)
## End(Not run)</pre>
```

28 iterativeMCMC class

iterativeMCMC class iterativeMCMC class structure

Description

The structure of an object of S3 class iterativeMCMC.

Details

An object of class iterativeMCMC is a list containing at least the following components:

- DAG: adjacency matrix of a maximum scoring DAG found/sampled in MCMC.
- CPDAG: adjacency matrix representing equivalence class of a maximum scoring DAG found/sampled in MCMC.
- score: score of a maximum scoring DAG found/sampled in MCMC.
- maxorder: order of a maximum scoring DAG found/sampled in MCMC.
- maxtrace: a list of maximum score graphs uncovered at each expansion of the search space; their scores and orders
- info: a list containing information about parameters and results of MCMC
- trace: a list of vectors containing log-scores of sampled DAGs, each element of the list corresponds to a single expansion of a search space
- startspace: adjacency matrix representing the initial core space where MCMC was ran
- endspace: adjacency matrix representing the final core space where MCMC was ran Optional components:
 - traceadd: list which consists of three elements:
 - * incidence: list containg adjacency matrices of sampled DAGs
 - * order: list of orders from which the DAGs were sampled
 - * orderscores: a list of vectors with order log-scores
 - scoretable: object of class scorespace class

Author(s)

Polina Suter

itercomp 29

itercomp	Performance assessment of iterative MCMC scheme against a known Bayesian network

Description

This function compute 8 different metrics of structure fit of an object of class iterativeMCMC to the ground truth DAG (or CPDAG). Object of class iterativeMCMC stores MAP graph at from each search space expansion step. This function computes structure fit of each of the stored graphs to the ground truth one. Computed metrics include: TP, FP, TPR, FPRn, FDR, SHD. See metrics description in see also compareDAGs.

Usage

```
itercomp(MCMCmult, truedag, cpdag = TRUE, p = 0.5, trans = TRUE)
## S3 method for class 'itercomp'
plot(x, ..., vars = c("FP", "TP"), type = "b", col = "blue", showit = c())
## S3 method for class 'itercomp'
print(x, ...)
## S3 method for class 'itercomp'
summary(object, ...)
```

Arguments

MCMCmult	an object which of class iterativeMCMC, see also iterativeMCMC)
truedag	ground truth DAG which generated the data used in the search procedure; represented by an object of class graphNEL or an adjacency matrix
cpdag	logical, if TRUE (FALSE by default) all DAGs are first converted to their respective equivalence classes (CPDAG)
p	threshold such that only edges with a higher posterior probability will be retained in the directed graph summarising the sample of DAGs at each iteration from MCMCmult if parameter sample set to TRUE
trans	logical, for DBNs indicates if model comparions are performed for transition structure; when trans equals FALSE the comparison is performed for initial structures of estimated models and the ground truth DBN; for usual BNs the parameter is disregarded
X	object of class 'itercomp'
	ignored
vars	a tuple of variables which will be used for 'x' and 'y' axes; possible values: "SHD", "TP", "FP", "TPR", "FPR", "FPRn", "FDR", "score"
type	type of line in the plot;"b" by default

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col colour of line in the plot; "blue" by default

showit (optional) vector of integers specifying indices of search expansion iterations to

be labelled; by default no iterations are labelled

object of class 'itercomp'

Value

an object if class itersim, a matrix with the number of rows equal to the number of expansion iterations in iterativeMCMC, and 8 columns reporting for the maximally scoring DAG uncovered at each iteration: the number of true positive edges ('TP'), the number of false positive edges ('FP'), the true positive rate ('TPR'), the structural Hamming distance ('SHD'), false positive rate ('FPR'), false discovery rate ('FDR') and the score of the DAG ('score').

Author(s)

Polina Suter

Examples

```
gsim.score<-scoreparameters("bge", gsim)
## Not run:
MAPestimate<-learnBN(gsim.score,"orderIter")
itercomp(MAPestimate, gsimmat)
## End(Not run)</pre>
```

kirc

kirc dataset

Description

Mutation data from TCGA kidney renal clear cell cohort (KIRC). Mutations are picked according to q-value computed by MutSig2CV (q<0.1) or connected in networks discovered by Kuipers et al. 2018.

Usage

kirc

Format

An object of class matrix (inherits from array) with 476 rows and 70 columns.

Details

Each variable represents a gene. If in sample i gene j contains a mutation, than j-th element in row i equals 1, and 0 otherwise. The rows are named according to sample names in TCGA. The columns are named according to gene symbols.

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References

```
https://portal.gdc.cancer.gov/
```

http://firebrowse.org/iCoMut/?cohort=kirc

Lawrence, M. et al. Mutational heterogeneity in cancer and the search for new cancer-associated genes. Nature 499, 214-218 (2013)

kirp

kirp dataset

Description

Mutation data from TCGA kidney renal papillary cell cohort (KIRP). Mutations are picked according to q-value computed by MutSigCV (q<0.1) or connected in networks discovered by Kuipers et al. 2018.

Usage

kirp

Format

An object of class matrix (inherits from array) with 282 rows and 70 columns.

Details

Each variable represents a gene. If in sample i gene j contains a mutation, than j-th element in row i equals 1, and 0 otherwise. The rows are named according to sample names in TCGA. The columns are named according to gene symbols.

References

```
https://portal.gdc.cancer.gov/
```

http://firebrowse.org/iCoMut/?cohort=kirp

Lawrence, M. et al. Mutational heterogeneity in cancer and the search for new cancer-associated genes. Nature 499, 214-218 (2013)

32 learnBN

learnBN

Bayesian network structure learning

Description

This function can be used finding the maximum a posteriori (MAP) DAG using stochastic search relying on MCMC schemes. Due to the superexponential size of the search space, it must be reduced. By default the search space is limited to the skeleton found through the PC algorithm by means of conditional independence tests (using the functions skeleton and pc from the 'pcalg' package [Kalisch et al, 2012]). It is also possible to define an arbitrary search space by inputting an adjacency matrix, for example estimated by partial correlations or other network algorithms. Order MCMC scheme (algorithm="order") performs the search of a maximum scoring order and selects a maximum scoring DAG from this order as MAP. To avoid discovering a suboptimal graph due to the absence of some of the true positive edges in the search space, the function includes the possibility to expand the default or input search space, by allowing each node in the network to have one additional parent (plus1="TRUE"). This offers improvements in the learning of Bayesian networks. The iterative MCMC (algorithm="orderIter") scheme allows for iterative expansions of the search space. This is useful in cases when the initial search space is poor in a sense that it contains only a limited number of true positive edges. Iterative expansions of the search space efficiently solve this issue. However this scheme requires longer runtimes due to the need of running multiple consecutive MCMC chains. This function is a wrapper for the individual structure learning functions that implement each of the described algorithms; for details see orderMCMC, and iterativeMCMC.

Usage

```
learnBN(
  scorepar,
  algorithm = c("order", "orderIter"),
  chainout = FALSE,
  scoreout = ifelse(algorithm == "orderIter", TRUE, FALSE),
  alpha = 0.05,
 moveprobs = NULL,
  iterations = NULL,
  stepsave = NULL,
  gamma = 1,
  verbose = FALSE,
  compress = TRUE,
  startspace = NULL,
  blacklist = NULL,
  scoretable = NULL,
  startpoint = NULL,
  plus1 = TRUE,
 iterpar = list(softlimit = 9, mergetype = "skeleton", accum = FALSE, plus1it = NULL,
    addspace = NULL, alphainit = NULL),
  cpdag = FALSE,
  hardlimit = 12
```

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)

Arguments

scorepar an object of class scoreparameters, containing the data and score parameters, see constructor function scoreparameters algorithm MCMC scheme to be used for MAP structure learning; possible options are "order" (orderMCMC) or "orderIter" (iterativeMCMC) chainout logical, if TRUE the saved MCMC steps are returned, TRUE by default logical, if TRUE the search space and score tables are returned; FALSE by descoreout fault for "order", TRUE for "orderIter" alpha numerical significance value in $\{0,1\}$ for the conditional independence tests at the PC algorithm stage a numerical vector of 4 (for "order" and "orderIter" algorithms) or 5 values (for moveprobs "partition" algorithm) representing probabilities of the different moves in the space of order and partitions accordingly. The moves are described in the corresponding algorithm specific functions orderMCMC and partitionMCMC integer, the number of MCMC steps, the default value is $6n^2 \log n$ orderMCMC, iterations $20n^2 \log n$ for partitionMCMC and $3.5n^2 \log n$ for iterativeMCMC; where n is the number of nodes in the Bayesian network integer, thinning interval for the MCMC chain, indicating the number of steps stepsave between two output iterations, the default is iterations/1000 tuning parameter which transforms the score by raising it to this power, 1 by gamma default logical, if TRUE messages about the algorithm's progress will be printed, FALSE verbose by default logical, if TRUE adjacency matrices representing sampled graphs will be stored compress as a sparse Matrix (recommended); TRUE by default startspace (optional) a square sparse or ordinary matrix, of dimensions equal to the number of nodes, which defines the search space for the order MCMC in the form of an adjacency matrix. If NULL, the skeleton obtained from the PC-algorithm will be used. If startspace[i,j] equals to 1 (0) it means that the edge from node i to node j is included (excluded) from the search space. To include an edge in both directions, both startspace[i, j] and startspace[j, i] should be 1. blacklist (optional) a square sparse or ordinary matrix, of dimensions equal to the number of nodes, which defines edges to exclude from the search space. If blacklist[i,j] equals to 1 it means that the edge from node i to node j is excluded from the search space. (optional) object of class scorespace containing list of score tables calculated scoretable for example by the last iteration of the function iterativeMCMC. When not NULL, parameter startspace is ignored. startpoint (optional) integer vector of length n (representing an order when algorithm="order" or algorithm="orderIter") or an adjacency matrix or sparse adjacency matrix (representing a DAG when algorithm="partition"), which will be used as the starting point in the MCMC algorithm, the default starting point is random

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plus1 logical, if TRUE (default) the search is performed on the extended search space; only changable for orderMCMC; for other algorithms is fixed to TRUE iterpar addition list of parameters for the MCMC scheme implemeting iterative expansions of the search space; for more details see iterativeMCMC; list(posterior = 0.5, softlimit = 9, mergetype = "skeleton", accum = FALSE, plus1it = NULL, addspace = NULL, alphainit = NULL) logical, if TRUE the CPDAG returned by the PC algorithm will be used as the cpdag search space, if FALSE (default) the full undirected skeleton will be used as the

search space

hardlimit integer, limit on the size of parent sets in the search space; by default 14 when

MAP=TRUE and 20 when MAP=FALSE

Value

Depending on the value or the parameter algorithm returns an object of class orderMCMC or iterativeMCMC which contains log-score trace of sampled DAGs as well as adjacency matrix of the maximum scoring DAG(s), its score and the order or partition score. The output can optionally include DAGs sampled in MCMC iterations and the score tables. Optional output is regulated by the parameters chainout and scoreout. See orderMCMC class, iterativeMCMC class for a detailed description of the classes' structures.

Note

see also extractor functions getDAG, getTrace, getSpace, getMCMCscore.

Author(s)

Polina Suter, Jack Kuipers, the code partly derived from the order MCMC implementation from Kuipers J, Moffa G (2017) <doi:10.1080/01621459.2015.1133426>

References

P. Suter, J. Kuipers, G. Moffa, N.Beerenwinkel (2023) <doi:10.18637/jss.v105.i09>

Friedman N and Koller D (2003). A Bayesian approach to structure discovery in bayesian networks. Machine Learning 50, 95-125.

Kalisch M, Maechler M, Colombo D, Maathuis M and Buehlmann P (2012). Causal inference using graphical models with the R package pealg. Journal of Statistical Software 47, 1-26.

Geiger D and Heckerman D (2002). Parameter priors for directed acyclic graphical models and the characterization of several probability distributions. The Annals of Statistics 30, 1412-1440.

Kuipers J, Moffa G and Heckerman D (2014). Addendum on the scoring of Gaussian acyclic graphical models. The Annals of Statistics 42, 1689-1691.

Spirtes P, Glymour C and Scheines R (2000). Causation, Prediction, and Search, 2nd edition. The MIT Press.

m2graph 35

Examples

```
## Not run:
myScore<-scoreparameters("bge",Boston)
mapfit<-learnBN(myScore,"orderIter")
summary(mapfit)
plot(mapfit)
## End(Not run)</pre>
```

m2graph

Deriving a graph from an adjacancy matrix

Description

This function derives a graph object corresponding to an adjacency matrix

Usage

```
m2graph(adj, nodes = NULL)
```

Arguments

adj square adjacency matrix with elements in {0,1}, representing a graph

nodes (optional) labels of the nodes, c(1:n) are used by default

Value

object of class graphNEL (package 'graph'); if element adj[i,j] equals 1, then there is a directed edge from node i to node j in the graph, and no edge otherwise

Examples

```
m2graph(Asiamat)
```

mapping

mapping dataset

Description

A data frame containing mapping between names of genes used in kirp/kirc data sets and names used in STRING interactions list (see interactions).

Usage

mapping

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Format

A data frame with 46 rows and two columns:

- queryItem character, name used for structure learning
- preferredName character, name used in STRING interactions data set

Source

https://string-db.org/

modelp

Estimating a graph corresponding to a posterior probability threshold

Description

This function constructs a directed graph (not necessarily acyclic) including all edges with a posterior probability above a certain threshold. The posterior probability is evaluated as the Monte Carlo estimate from a sample of DAGs obtained via an MCMC scheme.

Usage

```
modelp(MCMCchain, p, pdag = FALSE, burnin = 0.2)
```

Arguments

MCMCchain	object of class partitionMCMC, orderMCMC or iterativeMCMC, representing the output of structure sampling function partitionMCMC or orderMCMC (the latter when parameter chainout=TRUE;
р	threshold such that only edges with a higher posterior probability will be retained in the directed graph summarising the sample of DAGs
pdag	logical, if TRUE (FALSE by default) all DAGs in the MCMCchain are first converted to equivalence class (CPDAG) before the averaging
burnin	number between 0 and 1, indicates the percentage of the samples which will be the discarded as 'burn-in' of the MCMC chain; the rest of the samples will be used to calculate the posterior probabilities; 0.2 by default

Value

a square matrix with dimensions equal to the number of variables representing the adjacency matrix of the directed graph summarising the sample of DAGs

Author(s)

Polina Suter

orderMCMC 37

Examples

```
Bostonscore<-scoreparameters("bge", Boston)
## Not run:
partfit<-sampleBN(Bostonscore, "partition")
hdag<-modelp(partfit, p=0.9)
## End(Not run)</pre>
```

orderMCMC

Structure learning with the order MCMC algorithm

Description

This function implements the order MCMC algorithm for the structure learning of Bayesian networks. This function can be used for MAP discovery and for sampling from the posterior distribution of DAGs given the data. Due to the superexponential size of the search space as the number of nodes increases, the MCMC search is performed on a reduced search space. By default the search space is limited to the skeleton found through the PC algorithm by means of conditional independence tests (using the functions skeleton and pc from the 'pcalg' package [Kalisch et al, 2012]). It is also possible to define an arbitrary search space by inputting an adjacency matrix, for example estimated by partial correlations or other network algorithms. Also implemented is the possibility to expand the default or input search space, by allowing each node in the network to have one additional parent. This offers improvements in the learning and sampling of Bayesian networks.

Usage

```
orderMCMC(
  scorepar,
 MAP = TRUE,
  plus1 = TRUE,
  chainout = FALSE,
  scoreout = FALSE,
 moveprobs = NULL,
  iterations = NULL,
  stepsave = NULL,
  alpha = 0.05,
  cpdag = FALSE,
  gamma = 1,
  hardlimit = ifelse(plus1, 14, 20),
  verbose = FALSE,
  compress = TRUE,
  startspace = NULL,
  blacklist = NULL,
  startorder = NULL,
  scoretable = NULL
```

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```
## S3 method for class 'orderMCMC'
plot(
    x,
    ...,
    burnin = 0.2,
    main = "DAG logscores",
    xlab = "iteration",
    ylab = "logscore",
    type = "l",
    col = "#0c2c84"
)

## S3 method for class 'orderMCMC'
print(x, ...)

## S3 method for class 'orderMCMC'
summary(object, ...)
```

Arguments

scorepar an object of class scoreparameters, containing the data and score parameters,

see constructor function scoreparameters

MAP logical, if TRUE (default) the search targets the MAP DAG (a DAG with max-

imum score), if FALSE at each MCMC step a DAG is sampled from the order

proportionally to its score

plus1 logical, if TRUE (default) the search is performed on the extended search space

chainout logical, if TRUE the saved MCMC steps are returned, TRUE by default

scoreout logical, if TRUE the search space and score tables are returned, FALSE by de-

fault

moveprobs a numerical vector of 4 values in $\{0, 1\}$ corresponding to the probabilities of the

following MCMC moves in the order space

• exchanging 2 random nodes in the order

• exchanging 2 adjacent nodes in the order

• placing a single node elsewhere in the order

· staying still

iterations integer, the number of MCMC steps, the default value is $6n^2 \log n$

stepsave integer, thinning interval for the MCMC chain, indicating the number of steps

between two output iterations, the default is iterations/1000

alpha numerical significance value in {0,1} for the conditional independence tests at

the PC algorithm stage

cpdag logical, if TRUE the CPDAG returned by the PC algorithm will be used as the

search space, if FALSE (default) the full undirected skeleton will be used as the

search space

gamma tuning parameter which transforms the score by raising it to this power, 1 by

default

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hardlimit	integer, limit on the size of parent sets in the search space; by default 14 when MAP=TRUE and 20 when MAP=FALSE
verbose	logical, if TRUE messages about the algorithm's progress will be printed, FALSE by default
compress	logical, if TRUE adjacency matrices representing sampled graphs will be stored as a sparse Matrix (recommended); TRUE by default
startspace	(optional) a square matrix, of dimensions equal to the number of nodes, which defines the search space for the order MCMC in the form of an adjacency matrix. If NULL, the skeleton obtained from the PC-algorithm will be used. If startspace[i,j] equals to 1 (0) it means that the edge from node i to node j is included (excluded) from the search space. To include an edge in both directions, both startspace[i,j] and startspace[j,i] should be 1.
blacklist	(optional) a square matrix, of dimensions equal to the number of nodes, which defines edges to exclude from the search space. If blacklist[i,j] equals to 1 it means that the edge from node i to node j is excluded from the search space.
startorder	(optional) integer vector of length n, which will be used as the starting order in the MCMC algorithm, the default order is random
scoretable	(optional) object of class scorespace containing list of score tables calculated for example by the last iteration of the function iterativeMCMC. When not NULL, parameter startspace is ignored.
x	object of class 'orderMCMC'
	ignored
burnin	number between 0 and 1, indicates the percentage of the samples which will be discarded as 'burn-in' of the MCMC chain; the rest of the samples will be used to calculate the posterior probabilities; 0.2 by default
main	name of the graph; "DAG logscores" by default
xlab	name of x-axis; "iteration"
ylab	name of y-axis; "logscore"
type	type of line in the plot; "l" by default
col	colour of line in the plot; "#0c2c84" by default
object	object of class 'orderMCMC'

Value

Object of class orderMCMC, which contains log-score trace of sampled DAGs as well as adjacency matrix of the maximum scoring DAG, its score and the order score. The output can optionally include DAGs sampled in MCMC iterations and the score tables. Optional output is regulated by the parameters chainout and scoreout. See orderMCMC class for a detailed class structure.

Note

see also extractor functions getDAG, getTrace, getSpace, getMCMCscore.

40 orderMCMC class

Author(s)

Polina Suter, Jack Kuipers, the code partly derived from the order MCMC implementation from Kuipers J, Moffa G (2017) <doi:10.1080/01621459.2015.1133426>

References

P. Suter, J. Kuipers, G. Moffa, N.Beerenwinkel (2023) <doi:10.18637/jss.v105.i09>

Friedman N and Koller D (2003). A Bayesian approach to structure discovery in bayesian networks. Machine Learning 50, 95-125.

Kalisch M, Maechler M, Colombo D, Maathuis M and Buehlmann P (2012). Causal inference using graphical models with the R package pealg. Journal of Statistical Software 47, 1-26.

Geiger D and Heckerman D (2002). Parameter priors for directed acyclic graphical models and the characterization of several probability distributions. The Annals of Statistics 30, 1412-1440.

Kuipers J, Moffa G and Heckerman D (2014). Addendum on the scoring of Gaussian acyclic graphical models. The Annals of Statistics 42, 1689-1691.

Spirtes P, Glymour C and Scheines R (2000). Causation, Prediction, and Search, 2nd edition. The MIT Press.

Examples

```
## Not run:
#find a MAP DAG with search space defined by PC and plus1 neighbourhood
Bostonscore<-scoreparameters("bge",Boston)
#estimate MAP DAG
orderMAPfit<-orderMCMC(Bostonscore)
summary(orderMAPfit)
#sample DAGs from the posterior distribution
ordersamplefit<-orderMCMC(Bostonscore,MAP=FALSE,chainout=TRUE)
plot(ordersamplefit)
## End(Not run)</pre>
```

orderMCMC class

orderMCMC class structure

Description

The structure of an object of S3 class orderMCMC.

Details

An object of class orderMCMC is a list containing at least the following components:

- DAG: adjacency matrix of a maximum scoring DAG found/sampled in the MCMC scheme.
- CPDAG: adjacency matrix representing equivalence class of a maximum scoring DAG found/sampled in MCMC.

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- score: score of a maximum scoring DAG found/sampled in MCMC.
- maxorder: order of a maximum scoring DAG found/sampled in MCMC.
- info: a list containing information about parameters and results of MCMC.
- trace: a vector containing log-scores of sampled DAGs.
 Optional components:
 - traceadd: list which consists of three or four elements (depending on MCMC scheme used for sampling):
 - * incidence: list containg adjacency matrices of sampled DAGs
 - * order: list of orders from which the DAGs were sampled
 - * orderscores: order log-scores
 - scoretable: object of class scorespace class

Author(s)

Polina Suter

partitionMCMC

DAG structure sampling with partition MCMC

Description

This function implements the partition MCMC algorithm for the structure learning of Bayesian networks. This procedure provides an unbiased sample from the posterior distribution of DAGs given the data. The search space can be defined either by a preliminary run of the function iterativeMCMC or by a given adjacency matrix (which can be the full matrix with zero on the diagonal, to consider the entire space of DAGs, feasible only for a limited number of nodes).

Usage

```
partitionMCMC(
   scorepar,
   moveprobs = NULL,
   iterations = NULL,
   stepsave = NULL,
   alpha = 0.05,
   gamma = 1,
   verbose = FALSE,
   scoreout = FALSE,
   compress = TRUE,
   startspace = NULL,
   blacklist = NULL,
   scoretable = NULL,
   startDAG = NULL
)
```

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```
## S3 method for class 'partitionMCMC'
plot(
    x,
    ...,
    burnin = 0.2,
    main = "DAG logscores",
    xlab = "iteration",
    ylab = "logscore",
    type = "1",
    col = "#0c2c84"
)

## S3 method for class 'partitionMCMC'
print(x, ...)

## S3 method for class 'partitionMCMC'
summary(object, ...)
```

Arguments

scorepar

an object of class scoreparameters, containing the data and scoring parameters; see constructor function scoreparameters.

moveprobs

(optional) a numerical vector of 5 values in $\{0,1\}$ corresponding to the following MCMC move probabilities in the space of partitions:

- swap any two elements from different partition elements
- · swap any two elements in adjacent partition elements
- split a partition element or join one
- move a single node into another partition element or into a new one
- stay still

iterations

integer, the number of MCMC steps, the default value is $20n^2 \log n$

stepsave

integer, thinning interval for the MCMC chain, indicating the number of steps

between two output iterations, the default is iterations/1000

alpha

numerical significance value in $\{0,1\}$ for the conditional independence tests at

the PC algorithm stage

gamma

tuning parameter which transforms the score by raising it to this power, 1 by

default

verbose

logical, if set to TRUE (default) messages about progress will be printed

scoreout

logical, if TRUE the search space and score tables are returned, FALSE by de-

fault

compress

logical, if TRUE adjacency matrices representing sampled graphs will be stored

as a sparse Matrix (recommended); TRUE by default

startspace

(optional) a square matrix, of dimensions equal to the number of nodes, which defines the search space for the order MCMC in the form of an adjacency matrix; if NULL, the skeleton obtained from the PC-algorithm will be used. If startspace[i,j] equals to 1 (0) it means that the edge from node i to node j

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	is included (excluded) from the search space. To include an edge in both directions, both $startspace[i,j]$ and $startspace[j,i]$ should be 1.
blacklist	(optional) a square matrix, of dimensions equal to the number of nodes, which defines edges to exclude from the search space; if $blacklist[i,j]=1$ it means that the edge from node i to node j is excluded from the search space
scoretable	(optional) object of class scorespace containing list of score tables calculated for example by the last iteration of the function iterativeMCMC. When not NULL, parameter startspace is ignored
startDAG	(optional) an adjacency matrix of dimensions equal to the number of nodes, representing a DAG in the search space defined by startspace. If startspace is defined but startDAG is not, an empty DAG will be used by default
x	object of class 'partitionMCMC'
	ignored
burnin	number between 0 and 1, indicates the percentage of the samples which will be discarded as 'burn-in' of the MCMC chain; the rest of the samples will be used to calculate the posterior probabilities; 0.2 by default
main	name of the graph; "DAG logscores" by default
xlab	name of x-axis; "iteration"
ylab	name of y-axis; "logscore"
type	type of line in the plot; "l" by default
col	colour of line in the plot; "#0c2c84" by default
object	object of class 'partitionMCMC'

Value

Object of class partitionMCMC, which contains log-score trace as well as adjacency matrix of the maximum scoring DAG, its score and the order score. Additionally, returns all sampled DAGs (represented by their adjacency matrices), their scores, orders and partitions See partitionMCMC class.

Note

see also extractor functions getDAG, getTrace, getSpace, getMCMCscore.

Author(s)

Jack Kuipers, Polina Suter, the code partly derived from the partition MCMC implementation from Kuipers J, Moffa G (2017) <doi:10.1080/01621459.2015.1133426>

References

P. Suter, J. Kuipers, G. Moffa, N.Beerenwinkel (2023) <doi:10.18637/jss.v105.i09>

Kuipers J and Moffa G (2017). Partition MCMC for inference on acyclic digraphs. Journal of the American Statistical Association 112, 282-299.

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Geiger D and Heckerman D (2002). Parameter priors for directed acyclic graphical models and the characterization of several probability distributions. The Annals of Statistics 30, 1412-1440.

Heckerman D and Geiger D (1995). Learning Bayesian networks: A unification for discrete and Gaussian domains. In Eleventh Conference on Uncertainty in Artificial Intelligence, pages 274-284.

Kalisch M, Maechler M, Colombo D, Maathuis M and Buehlmann P (2012). Causal inference using graphical models with the R package pealg. Journal of Statistical Software 47, 1-26.

Kuipers J, Moffa G and Heckerman D (2014). Addendum on the scoring of Gaussian directed acyclic graphical models. The Annals of Statistics 42, 1689-1691.

Examples

```
## Not run:
myScore<-scoreparameters("bge", Boston)
partfit<-partitionMCMC(myScore)
plot(partfit)
## End(Not run)</pre>
```

partitionMCMC class

partitionMCMC class structure

Description

The structure of an object of S3 class partitionMCMC.

Details

An object of class partitionMCMC is a list containing at least the following components:

- DAG: adjacency matrix of a maximum scoring DAG found/sampled in the MCMC scheme.
- CPDAG: adjacency matrix representing equivalence class of a maximum scoring DAG found/sampled in MCMC.
- score: score of a maximum scoring DAG found/sampled in MCMC.
- maxorder: order of a maximum scoring DAG found/sampled in MCMC.
- info: a list containing information about parameters and results of MCMC.
- trace: a vector containing log-scores of sampled DAGs.
 Optional components:
 - traceadd: list which consists of three or four elements (depending on MCMC scheme used for sampling):
 - * incidence: list containg adjacency matrices of sampled DAGs
 - * order: list of orders from which the DAGs were sampled
 - * partition: list of partition from which the DAGs were sampled
 - * partitionscores: partition log-scores
 - scoretable: object of class scorespace class

plot2in1 45

Author(s)

Polina Suter

plot2in1	Highlighting similarities between two graphs
•	

Description

This function plots nodes and edges from two graphs in one and indicates similarities between these graphs.

Usage

```
plot2in1(graph1, graph2, name1 = NULL, name2 = NULL, bidir = FALSE, ...)
```

Arguments

graph1	binary adjacency matrix of a graph
graph2	binary adjacency matrix of a graph, column names should coincide with column names of 'graph1'
name1	character, custom name for 'graph1'; when NULL no legend will be plotted
name2	character, custom name for 'graph2'
bidir	logical, defines if arrows of bidirected edges are drawn; FALSE by defauls.
	optional parameters passed to Rgraphviz plotting functions e.g. main, fontsize

Value

plots the graph which includes nodes and edges two graphs; nodes which are connected to at least one other node in both graphs are plotted only once and coloured orange, edges which are shared by two graphs are coloured orange; all other nodes and edges a plotted once for each 'graph1' and 'graph2' and coloured blue and green accordingly.

Author(s)

Polina Suter

46 plotDBN

plotDBN	Plotting a DBN
---------	----------------

Description

This function can be used for plotting initial and transition structures of a dynamic Bayesian network.

Usage

```
plotDBN(DBN, struct = c("init", "trans"), b = 0, shape = "circle", ...)
```

Arguments

DBN	binary matrix (or a graph object) representing a 2-step DBN (compact or unrolled)
struct	option used to determine if the initial or the transition structure should be plotted; acceptable values are init or trans
b	number of static variables in the DBN, 0 by default; note that for function to work correctly all static variables have to be in the first b columns of the matrix
shape	string, defining the shape of the box around each node; possible values are circle, ellipse, box
	optional parameters passed to Rgraphviz plotting functions e.g. main, fontsize

Value

plots the DBN defined by the adjacency matrix 'DBN' and number of static and dynamic variables. When 'struct' equals "trans" the transition structure is plotted, otherwise initial structure is plotted

Author(s)

Polina Suter

Examples

```
plotDBN(DBNmat, "init", b=3)
plotDBN(DBNmat, "trans", b=3)
```

plotdiffs 47

plotdiffs	Plotting difference between two graphs	

Description

This function plots edges from two graphs in one and indicates similarities and differences between these graphs. It is also possible to use this function for plotting mistakes in estimated graph when the ground truth graph is known.

Usage

```
plotdiffs(
  graph1,
  graph2,
  estimated = TRUE,
  name1 = "graph1",
  name2 = "graph2",
  clusters = NULL,
  ...
)
```

Arguments

graph1	object of class graphNEL or its adjacency matrix
graph2	object of class graphNEL or its adjacency matrix
estimated	logical, indicates if graph1 is estimated graph and graph2 is ground truth DAG, TRUE by default; this affects the legend and colouring of the edges
name1	character, custom name for 'graph1'
name2	character, custom name for 'graph2'
clusters	(optional) a list of nodes to be represented on the graph as clusters
	optional parameters passed to Rgraphviz plotting functions e.g. main, fontsize

Value

plots the graph which includes edges from graph1 and graph2; edges which are different in graph1 compared to graph2 are coloured according to the type of a difference

Author(s)

Polina Suter

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Examples

```
Asiascore<-scoreparameters("bde",Asia)
Asiamap<-orderMCMC(Asiascore)
plotdiffs(Asiamap$DAG,Asiamat)
Asiacp<-pcalg::dag2cpdag(m2graph(Asiamat))
mapcp<-pcalg::dag2cpdag(m2graph(Asiamap$DAG))
plotdiffs(mapcp,Asiacp)
```

 ${\tt plotdiffsDBN}$

Plotting difference between two DBNs

Description

This function plots an estimated DBN such that the edges which are different to the ground truth DBN are highlighted.

Usage

```
plotdiffsDBN(
  eDBN,
  trueDBN,
  struct = c("init", "trans"),
  b = 0,
  showcl = TRUE,
  orientation = "TB",
  ...
)
```

Arguments

eDBN	object of class graphNEL (or its adjacency matrix), representing estimated structure (not necessarily acyclic) to be compared to the ground truth graph
trueDBN	object of class graphNEL (or its adjacency matrix), representing the ground truth structure (not necessarily acyclic)
struct	option used to determine if the initial or the transition structure should be plotted; accaptable values are init or trans
b	number of static variables in one time slice of a DBN; note that for function to work correctly all static variables have to be in the first b columns of the matrix
showc1	logical, when TRUE (default) nodes are shown in clusters according to the time slice the belong to
orientation	orientation of the graph layout, possible options are 'TB' (top-bottom) and 'LR' (left-right) $$
	optional parameters passed to Rgraphviz plotting functions e.g. main, fontsize

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Value

plots the graph highlights differences between 'eDBN' (estimated DBN) and 'trueDBN' (ground truth); edges which are different in 'eDBN' compared to 'trueDBN' are coloured according to the type of a difference: false-positive, false-negative and error in direction.

Author(s)

Polina Suter

Examples

```
dbnscore<-scoreparameters("bge",DBNdata,
dbnpar = list(samestruct=TRUE, slices=5, b=3),
DBN=TRUE)
## Not run:
orderDBNfit<-learnBN(dbnscore,algorithm="order")
iterDBNfit<-learnBN(dbnscore,algorithm="orderIter")
plotdiffsDBN(getDAG(orderDBNfit),DBNmat,struct="trans",b=3)
plotdiffsDBN(getDAG(iterDBNfit),DBNmat,struct="trans",b=3)</pre>
## End(Not run)
```

plotpcor

Comparing posterior probabilitites of single edges

Description

This function can be used to compare posterior probabilities of edges in a graph

equals TRUE

Usage

```
plotpcor(pmat, highlight = 0.3, printedges = FALSE, cut = 0.05, ...)
```

Arguments

pmat	a list of square matrices, representing posterior probabilities of single edges in a Bayesian network; see edgep for obtaining such a matrix from a single MCMC run
highlight	numeric, defines maximum acceptable difference between posterior probabilities of an edge in two samples; points corresponding to higher differences are highlighted in red
printedges	when TRUE the function also returns squared correlation and RMSE of posterior probabilities higher than the value defined by the argument 'cut' as well as the list of all edges whose posterior probabilities in the first two matrices differ more than 'highlight'; FALSE by default
cut	numeric value corresponding to a minimum posterior probabilitity which is included into calculation of squared correlation and MSE when 'printedges'

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... prameters passed further to the plot function (e.g. xlab, ylab, main) in case when the length of pmat equals 2

Value

plots concordance of posterior probabilities of single edges based on several matrices (minimum 2 matrices); highlights the edges whose posterior probabilities in a pair of matrices differ by more than 'highlight'; when 'printedges' set to TRUE, the function returns also squared correlation and RMSE of posterior probabilities higher than the value defined by the argument 'cut' as well as the list of all edges whose posterior probabilities in the first two matrices differ by more than 'highlight'.

Author(s)

Polina Suter

Examples

```
Asiascore<-scoreparameters("bde", Asia)
## Not run:
orderfit<-list()
orderfit[[1]]<-sampleBN(Asiascore,algorithm="order")
orderfit[[2]]<-sampleBN(Asiascore,algorithm="order")
orderfit[[3]]<-sampleBN(Asiascore,algorithm="order")
pedges<-lapply(orderfit,edgep,pdag=TRUE)
plotpcor(pedges, xlab="run1", ylab="run2",printedges=TRUE)
## End(Not run)
```

plotpedges

Plotting posterior probabilities of single edges

Description

This function plots posterior probabilities of all possible edges in the graph as a function of MCMC iterations. It can be used for convergence diagnostics of MCMC sampling algorithms order MCMC and partition MCMC.

Usage

```
plotpedges(
   MCMCtrace,
   cutoff = 0.2,
   pdag = FALSE,
   onlyedges = NULL,
   highlight = NULL,
   ...
)
```

Arguments

MCMCtrace an object of class MCMCres

cutoff number representing a threshold of posterior probability below which lines will

not be plotted

pdag logical, when true DAGs in a sample will be first coverted to CPDAGs

onlyedges (optional) binary matrix, only edges corresponding to entries which equal 1 will

be plotted

highlight (optional) binary matrix, edges corresponding to entries which equal 1 are high-

lighted with "red"

... (optional) parameters passed to the plot function

Value

plots posterior probabilities of edges in the graph as a function of MCMC iterations

Author(s)

Polina Suter

Examples

```
score100<-scoreparameters("bde", Asia[1:100,])
orderfit100<-orderMCMC(score100,plus1=TRUE,chainout=TRUE)
## Not run:
score5000<-scoreparameters("bde", Asia)
orderfit5000<-orderMCMC(score5000,plus1=TRUE,chainout=TRUE)
plotpedges(orderfit100, pdag=TRUE)
plotpedges(orderfit5000, pdag=TRUE)
## End(Not run)</pre>
```

sampleBN

Bayesian network structure sampling from the posterior distribution

Description

This function can be used for structure sampling using three different MCMC schemes. Order MCMC scheme (algorithm="order") is the most computationally efficient however it imposes a non-uniform prior in the space of DAGs. Partition MCMC (algorithm="partition") is less computationally efficient and requires more iterations to reach convergence, however it implements sampling using a uniform prior in the space of DAGs. Due to the superexponential size of the search space as the number of nodes increases, the MCMC search is performed on a reduced search space. By default the search space is limited to the skeleton found through the PC algorithm by means of conditional independence tests (using the functions skeleton and pc from the 'pcalg' package [Kalisch et al, 2012]). It is also possible to define an arbitrary search space by inputting an adjacency matrix, for example estimated by partial correlations or other network algorithms.

Also implemented is the possibility to expand the default or input search space, by allowing each node in the network to have one additional parent. This offers improvements in the learning and sampling of Bayesian networks. The iterative MCMC scheme (algorithm="orderIter") allows for iterative expansions of the search space. This is useful in cases when the initial search space is poor in a sense that it contains only a limited number of true positive edges. Iterative expansions of the search space efficiently solve this issue. However this scheme requires longer runtimes due to the need of running multiple consecutive MCMC chains. This function is a wrapper for the three individual structure learning and sampling functions that implement each of the described algorithms; for details see orderMCMC, partitionMCMC, iterativeMCMC.

Usage

```
sampleBN(
  scorepar,
  algorithm = c("order", "orderIter", "partition"),
  chainout = TRUE,
  scoreout = FALSE,
  alpha = 0.05,
 moveprobs = NULL,
  iterations = NULL,
  stepsave = NULL,
  gamma = 1,
  verbose = FALSE,
  compress = TRUE,
  startspace = NULL,
 blacklist = NULL,
  scoretable = NULL,
  startpoint = NULL,
  plus1 = TRUE,
  cpdag = FALSE,
  hardlimit = 12,
 iterpar = list(posterior = 0.5, softlimit = 9, mergetype = "skeleton", accum = FALSE,
    plus1it = NULL, addspace = NULL, alphainit = NULL)
)
```

Arguments

scorepar	see constructor function scoreparameters, containing the data and score parameters,
algorithm	MCMC scheme to be used for sampling from posterior distribution; possible options are "order" (orderMCMC), "orderIter" (iterativeMCMC) or "partition" (partitionMCMC)
chainout	logical, if TRUE the saved MCMC steps are returned, TRUE by default
scoreout	logical, if TRUE the search space and score tables are returned, FALSE by default
alpha	numerical significance value in $\{0,1\}$ for the conditional independence tests at the PC algorithm stage

an object of class comparameters, containing the data and score parameters

a numerical vector of 4 (for "order" and "orderIter" algorithms) or 5 values (for moveprobs "partition" algorithm) representing probabilities of the different moves in the space of order and partitions accordingly. The moves are described in the corresponding algorithm specific functions orderMCMC and partitionMCMC iterations integer, the number of MCMC steps, the default value is $6n^2 \log n$ orderMCMC, $20n^2 \log n$ for partitionMCMC and $3.5n^2 \log n$ for iterativeMCMC; where n is the number of nodes in the Bayesian network stepsave integer, thinning interval for the MCMC chain, indicating the number of steps between two output iterations, the default is iterations/1000 gamma tuning parameter which transforms the score by raising it to this power, 1 by default verbose logical, if TRUE messages about the algorithm's progress will be printed, FALSE by default logical, if TRUE adjacency matrices representing sampled graphs will be stored compress as a sparse Matrix (recommended); TRUE by default startspace (optional) a square sparse or ordinary matrix, of dimensions equal to the number of nodes, which defines the search space for the order MCMC in the form of an adjacency matrix. If NULL, the skeleton obtained from the PC-algorithm will be used. If startspace[i, j] equals to 1 (0) it means that the edge from node i to node j is included (excluded) from the search space. To include an edge in both directions, both startspace[i,j] and startspace[j,i] should be 1. blacklist (optional) a square sparse or ordinary matrix, of dimensions equal to the number of nodes, which defines edges to exclude from the search space. If blacklist[i,j] equals to 1 it means that the edge from node i to node j is excluded from the search space. (optional) object of class scorespace containing list of score tables calculated scoretable for example by the last iteration of the function iterativeMCMC. When not NULL, parameter startspace is ignored. (optional) integer vector of length n (representing an order when algorithm="order" startpoint or algorithm="orderIter") or an adjacency matrix or sparse adjacency matrix (representing a DAG when algorithm="partition"), which will be used as the starting point in the MCMC algorithm, the default starting point is random plus1 logical, if TRUE (default) the search is performed on the extended search space; only changable for orderMCMC; for other algorithms is fixed to TRUE logical, if TRUE the CPDAG returned by the PC algorithm will be used as the cpdag search space, if FALSE (default) the full undirected skeleton will be used as the search space hardlimit integer, limit on the size of parent sets in the search space; iterpar addition list of parameters for the MCMC scheme implemeting iterative expansions of the search space; for more details see iterativeMCMC; list(posterior = 0.5, softlimit = 9, mergetype = "skeleton", accum = FALSE, plus1it = NULL, addspace = NULL, alphainit = NULL)

Value

Depending on the value or the parameter algorithm returns an object of class orderMCMC, partitionMCMC or iterativeMCMC which contains log-score trace of sampled DAGs as well as adjacency matrix of the maximum scoring DAG(s), its score and the order or partition score. The output can optionally include DAGs sampled in MCMC iterations and the score tables. Optional output is regulated by the parameters chainout and scoreout. See orderMCMC class, partitionMCMC class, iterativeMCMC class for a detailed description of the classes' structures.

Note

see also extractor functions getDAG, getTrace, getSpace, getMCMCscore.

Author(s)

Polina Suter, Jack Kuipers, the code partly derived from the order MCMC implementation from Kuipers J, Moffa G (2017) <doi:10.1080/01621459.2015.1133426>

References

P. Suter, J. Kuipers, G. Moffa, N.Beerenwinkel (2023) <doi:10.18637/jss.v105.i09>

Friedman N and Koller D (2003). A Bayesian approach to structure discovery in bayesian networks. Machine Learning 50, 95-125.

Kalisch M, Maechler M, Colombo D, Maathuis M and Buehlmann P (2012). Causal inference using graphical models with the R package pealg. Journal of Statistical Software 47, 1-26.

Geiger D and Heckerman D (2002). Parameter priors for directed acyclic graphical models and the characterization of several probability distributions. The Annals of Statistics 30, 1412-1440.

Kuipers J, Moffa G and Heckerman D (2014). Addendum on the scoring of Gaussian acyclic graphical models. The Annals of Statistics 42, 1689-1691.

Spirtes P, Glymour C and Scheines R (2000). Causation, Prediction, and Search, 2nd edition. The MIT Press.

Examples

```
## Not run:
Asiascore <- scoreparameters("bde", Asia)
iterativefit <- learnBN(Asiascore, algorithm = "orderIter")
orderfit <- sampleBN(Asiascore, scoretable = iterativefit)

myScore<-scoreparameters("bge",Boston)
MCMCchains<-list()
MCMCchains[[1]]<-sampleBN(myScore,"partition")
MCMCchains[[2]]<-sampleBN(myScore,"partition")
edge_posterior<-lapply(MCMCchains,edgep,pdag=TRUE)
plotpcor(edge_posterior)

## End(Not run)</pre>
```

55 samplecomp

samplecomp	Performance assessment of sampling algorithms against a known
	Bayesian network

Description

This function compute 8 different metrics of structure fit of an object of classes orderMCMC and partitionMCMC to the ground truth DAG (or CPDAG). First posterior probabilities of single edges are calculated based on a sample stores in the object of class orderMCMC or partitionMCMC. This function computes structure fit of each of the consensus graphs to the ground truth one based on a defined range of posterior thresholds. Computed metrics include: TP, FP, TPR, FPR, FPRn, FDR, SHD. See metrics description in see also compareDAGs.

Usage

```
samplecomp(
 MCMCchain,
  truedag,
  p = c(0.99, 0.95, 0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3, 0.2),
  pdag = TRUE,
 burnin = 0.2,
  trans = TRUE
)
## S3 method for class 'samplecomp'
plot(x, ..., vars = c("FP", "TP"), type = "b", col = "blue", showp = NULL)
## S3 method for class 'samplecomp'
print(x, ...)
## S3 method for class 'samplecomp'
summary(object, ...)
```

equivalence class (CPDAG) before the averaging

Arguments

MCMCchain	an object of class partitionMCMC or orderMCMC, representing the output of structure sampling function partitionMCMC or orderMCMC (the latter when parameter chainout=TRUE;
truedag	ground truth DAG which generated the data used in the search procedure; represented by an object of class graphNEL
p	a vector of numeric values between 0 and 1, defining posterior probabilities according to which the edges of assessed structures are drawn, please note very low barriers can lead to very dense structures; by default $p=c(0.99,0.95,0.9,0.8,0.7,0.6,0.5,0.4,0.3,0.2)$
pdag	logical, if TRUE (default) all DAGs in the MCMCchain are first converted to

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burnin	number between 0 and 1, indicates the percentage of the samples which will be the discarded as 'burn-in' of the MCMC chain; the rest of the samples will be used to calculate the posterior probabilities; 0.2 by default
trans	logical, for DBNs indicates if model comparions are performed for transition structure; when trans equals FALSE the comparison is performed for initial structures of estimated models and the ground truth DBN; for usual BNs the parameter is disregarded
x	object of class 'samplecomp'
	ignored
vars	a tuple of variables which will be used for 'x' and 'y' axes; possible values: "SHD", "TP", "FPR", "FPR", "FPRn", "FDR"
type	type of line in the plot; "b" by default
col	colour of line in the plotl; "blue" by default
showp	logical, defines if points are labelled with the posterior threshold corresponding to the assessed model
object	object of class 'samplecomp'

Value

an object if class samplesim, a matrix with the number of rows equal to the number of elements in 'p', and 8 columns reporting for the consensus graphss (corresponding to each of the values in 'p') the number of true positive edges ('TP'), the number of false positive edges ('FP'), the number of false negative edges ('FN'), the true positive rate ('TPR'), the structural Hamming distance ('SHD'), false positive rate ('FPR'), false discovery rate ('FDR') and false positive rate normalized by TP+FN ('FPRn').

Author(s)

Polina Suter

Examples

```
gsim.score<-scoreparameters("bge", gsim)
## Not run:
MAPestimate<-learnBN(gsim.score, "orderIter", scoreout=TRUE)
ordersample<-sampleBN(gsim.score, "order", scoretable=getSpace(MAPestimate))
samplecomp(ordersample, gsimmat)
## End(Not run)</pre>
```

scoreagainstDAG 57

calculating the score of a sample against a DAG

Description

This function calculates the score of a given sample against a DAG represented by its incidence matrix.

Usage

```
scoreagainstDAG(
   scorepar,
   incidence,
   datatoscore = NULL,
   marginalise = FALSE,
   onlymain = FALSE,
   bdecatCvec = NULL
)
```

Arguments

scorepar	an object of class scoreparameters; see constructor function scoreparameters
incidence	a square matrix of dimensions equal to the number of variables with entries in $\{0,1\}$, representing the adjacency matrix of the DAG against which the score is calculated
datatoscore	(optional) a matrix (vector) containing binary (for BDe score) or continuous (for the BGe score) observations (or just one observation) to be scored; the number of columns should be equal to the number of variables in the Bayesian network, the number of rows should be equal to the number of observations; by default all data from scorepar parameter is used
marginalise	(optional for continuous data) defines, whether to use the posterior mean for scoring (default) or to marginalise over the posterior distribution (more computationally costly)
onlymain	(optional), defines the the score is computed for nodes excluding 'bgnodes'; FALSE by default
bdecatCvec	(optional for categorical data)

Value

the log of the BDe/BGe score of given observations against a DAG

Author(s)

Jack Kuipers, Polina Suter

58 scoreagainstDBN

References

Heckerman D and Geiger D, (1995). Learning Bayesian networks: A unification for discrete and Gaussian domains. In Eleventh Conference on Uncertainty in Artificial Intelligence, pages 274-284, 1995.

Examples

Asiascore<-scoreparameters("bde", Asia[1:100,]) #we wish to score only first 100 observations scoreagainstDAG(Asiascore, Asiamat)

scoreagainstDBN

Score against DBN

Description

Scoring observations against a DBN structure

Usage

```
scoreagainstDBN(
  scorepar,
  incidence,
  datatoscore = NULL,
  marginalise = FALSE,
  onlymain = FALSE,
  datainit = NULL
)
```

Arguments

scorepar object of class 'scoreparameters' incidence adjacency matrix of a DAG

datatoscore matrix or vector containing observations to be scored

marginalise (logical) should marginal score be used?

onlymain (logical) should static nodes be included in the score?

datainit optional, in case of unbalanced design, the mean score of available samples for

T0 are computed

Value

vector of log-scores

Author(s)

Polina Suter

scoreparameters 59

scoreparameters

Initializing score object

Description

This function returns an object of class scoreparameters containing the data and parameters needed for calculation of the BDe/BGe score, or a user defined score.

Usage

```
scoreparameters(
  scoretype = c("bge", "bde", "bdecat", "usr"),
  data,
  bgepar = list(am = 1, aw = NULL, edgepf = 1),
 bdepar = list(chi = 0.5, edgepf = 2),
 bdecatpar = list(chi = 0.5, edgepf = 2),
 dbnpar = list(samestruct = TRUE, slices = 2, b = 0, stationary = TRUE, rowids = NULL,
    datalist = NULL, learninit = TRUE),
  usrpar = list(pctesttype = c("bge", "bde", "bdecat")),
 mixedpar = list(nbin = 0),
 MDAG = FALSE,
 DBN = FALSE,
 weightvector = NULL,
  bgnodes = NULL,
  edgepmat = NULL,
  nodeslabels = NULL
)
## S3 method for class 'scoreparameters'
print(x, ...)
## S3 method for class 'scoreparameters'
summary(object, ...)
```

Arguments

scoretype the score to be used to assess the DAG structure: "bge" for Gaussian data,

"bde" for binary data, "bdecat" for categorical data, "usr" for a user defined score; when "usr" score is chosen, one must define a function (which evaluates the log score of a node given its parents) in the following format: usrDAG-corescore(j,parentnodes,n,param), where 'j' is node to be scores, 'parentnodes' are the parents of this node, 'n' number of nodes in the network and 'param' is

an object of class 'scoreparameters'

data the data matrix with n columns (the number of variables) and a number of rows

equal to the number of observations

bgepar a list which contains parameters for BGe score:

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- am (optional) a positive numerical value, 1 by default
- aw (optional) a positive numerical value should be more than n+1, n+am+1 by default
- edgepf (optional) a positive numerical value providing the edge penalization factor to be combined with the BGe score, 1 by default (no penalization)

bdepar

a list which contains parameters for BDe score for binary data:

- chi (optional) a positive number of prior pseudo counts used by the BDe score, 0.5 by default
- edgepf (optional) a positive numerical value providing the edge penalization factor to be combined with the BDe score, 2 by default

bdecatpar

a list which contains parameters for BDe score for categorical data:

- chi (optional) a positive number of prior pseudo counts used by the BDe score, 0.5 by default
- edgepf (optional) a positive numerical value providing the edge penalization factor to be combined with the BDe score, 2 by default

dbnpar

which type of score to use for the slices

- samestruct logical, when TRUE the structure of the first time slice is assumed to be the same as internal structure of all other time slices
- slices integer representing the number of time slices in a DBN
- b the number of static variables; all static variables have to be in the first b columns of the data; for DBNs static variables have the same meaning as bgnodes for usual Bayesian networks; for DBNs parameters parameter bgnodes is ignored
- rowids optional vector of time IDs; usefull for identifying data for initial time slice
- datalist indicates is data is passed as a list for a two step DBN; useful for unbalanced number of samples in timi slices

usrpar

a list which contains parameters for the user defined score

• pctesttype (optional) conditional independence test ("bde","bge","bdecat")

mixedpar

a list which contains parameters for the BGe and BDe score for mixed data

 nbin a positive integer number of binary nodes in the network (the binary nodes are always assumed in first nbin columns of the data)

MDAG

logical, when TRUE the score is initialized for a model with multiple sets of parameters but the same structure

DBN

logical, when TRUE the score is initialized for a dynamic Baysian network; FALSE by default

weightvector

(optional) a numerical vector of positive values representing the weight of each observation; should be NULL(default) for non-weighted data

bgnodes

(optional) a vector that contains column indices in the data defining the nodes that are forced to be root nodes in the sampled graphs; root nodes are nodes which have no parents but can be parents of other nodes in the network; in case of DBNs bgnodes represent static variables and defined via element b of the parameters dbnpar; parameter bgnodes is ignored for DBNs

scorespace 61

edgepmat	(optional) a matrix of positive numerical values providing the per edge penalization factor to be added to the score, NULL by default
nodeslabels	(optional) a vector of characters which denote the names of nodes in the Bayesian network; by default column names of the data will be taken
x	object of class 'scoreparameters'
	ignored
obiect	object of class 'scoreparameters'

Value

an object of class scoreparameters, which includes all necessary information for calculating the BDe/BGe score

Author(s)

Polina Suter, Jack kuipers

References

Geiger D and Heckerman D (2002). Parameter priors for directed acyclic graphical models and the characterization of several probability distributions. The Annals of Statistics 30, 1412-1440.

Kuipers J, Moffa G and Heckerman D (2014). Addendum on the scoring of Gaussian acyclic graphical models. The Annals of Statistics 42, 1689-1691.

Heckerman D and Geiger D (1995). Learning Bayesian networks: A unification for discrete and Gaussian domains. In Eleventh Conference on Uncertainty in Artificial Intelligence, pages 274-284.

Scutari M (2016). An Empirical-Bayes Score for Discrete Bayesian Networks. Journal of Machine Learning Research 52, 438-448

Examples

```
myDAG<-pcalg::randomDAG(20, prob=0.15, lB = 0.4, uB = 2)
myData<-pcalg::rmvDAG(200, myDAG)
myScore<-scoreparameters("bge", myData)</pre>
```

scorespace

Prints 'scorespace' object

Description

Prints 'scorespace' object

Summary of object of class 'scorespace'

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Usage

```
scorespace(
   scorepar,
   alpha = 0.05,
   hardlimit = 14,
   plus1 = TRUE,
   cpdag = TRUE,
   startspace = NULL,
   blacklist = NULL,
   verbose = FALSE
)

## S3 method for class 'scorespace'
print(x, ...)

## S3 method for class 'scorespace'
summary(object, ...)
```

Arguments

scorepar	an object of class scoreparameters.	containing the data and score scorepareters.

see constructor function scoreparameters

alpha numerical significance value in {0,1} for the conditional independence tests at

the PC algorithm stage (by default 0.4 for n < 50, 20/n for n > 50)

hardlimit integer, limit on the size of parent sets in the search space; by default 14 when

MAP=TRUE and 20 when MAP=FALSE

plus1 logical, if TRUE (default) the search is performed on the extended search space

cpdag logical, if TRUE the CPDAG returned by the PC algorithm will be used as the

search space, if FALSE (default) the full undirected skeleton will be used as the

search space

startspace (optional) a square matrix, of dimensions equal to the number of nodes, which

defines the search space for the order MCMC in the form of an adjacency matrix. If NULL, the skeleton obtained from the PC-algorithm will be used. If startspace[i,j] equals to 1 (0) it means that the edge from node i to node j is included (excluded) from the search space. To include an edge in both direc-

tions, both startspace[i,j] and startspace[j,i] should be 1.

blacklist (optional) a square matrix, of dimensions equal to the number of nodes, which

defines edges to exclude from the search space. If blacklist[i,j] equals to 1 it means that the edge from node i to node j is excluded from the search space.

verbose logical, if TRUE messages about the algorithm's progress will be printed, FALSE

by default

x object of class 'scorespace'

... ignored

object of class 'scorespace'

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Value

Object of class scorespace, a list of three objects: 'adjacency' matrix representiong the search space, 'blacklist' used to exclude edges from the search space and 'tables' containing score quantities for each node needed to run MCMC schemes

Author(s)

Polina Suter, Jack Kuipers

References

Friedman N and Koller D (2003). A Bayesian approach to structure discovery in bayesian networks. Machine Learning 50, 95-125.

Examples

```
#' #find a MAP DAG with search space defined by PC and plus1 neighbourhood
Bostonscore<-scoreparameters("bge",Boston)
Bostonspace<-scorespace(Bostonscore, 0.05, 14)
## Not run:
orderfit<-orderMCMC(Bostonscore, scoretable=Bostonspace)
partitionfit<-orderMCMC(Bostonscore, scoretable=Bostonspace)
## End(Not run)</pre>
```

scorespace class

scorespace class structure

Description

The structure of an object of S3 class scorespace.

Details

An object of class scorespace is a list containing at least the following components:

- adjacency: adjacency martrix representing the core search space
- · blacklist: adjacency martrix representing the blacklist used for computing score tables tables
- tables: a list of matrices (for core search space) or a list of lists of matrices (for extended search space) containing quantities needed for scoring orders and sampling DAGs in MCMC schemes; this list corresponds to adjacency and blacklist

Author(s)

Polina Suter

string2mat

|--|

Description

This transforms a list of possible interactions between proteins downloaded from STRING database into a matrix which can be used for blacklisting/penalization in BiDAG.

Usage

```
string2mat(curnames, int, mapping = NULL, type = c("int"), pf = 2)
```

Arguments

curnames	character vector with gene names which will be used in BiDAG learning function
int	data frame, representing a interactions between genes/proteins downloaded from STRING (https://string-db.org/); two columns are necessary 'node1' and 'node2'
mapping	(optional) data frame, representing a mapping between 'curnames' (gene names, usually the column names of 'data') and gene names used in interactions downloaded from STRING (https://string-db.org/); two columns are necessary 'queryItem' and 'preferredName'
type	character, defines how interactions will be reflected in the output matrix; int will result in a matrix whose entries equal 1 if interaction is present in the list of interactions int and 0 otherwise; blacklist results in a matrix whose entries equal 0 when interaction is present in the list of interactions and 1 otherwise; pf results in a matrix results in a matrix whose entries equal 1 is interaction is present in the list of interactions int and pf otherwise\$ "int" by default
pf	penalization factor for interactions, needed if type=pf

Value

square matrix whose entries correspond to the list of interactions and parameter type

Examples

```
curnames<-colnames(kirp)
intmat<-string2mat(curnames, mapping, interactions, type="pf")</pre>
```

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