# A Quick Guide for the phyclust Package

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This document is written to explain the major functions of **phyclust** according to version 0.1-12. Every effort will be made to insure future versions are consistent with these instructions, but new features in later versions may not be explained in this document.

### 1. Introduction

This is a quick guide to the package **phyclust** (Chen 2011) implementing model-based phylogenetic clustering aiming to cluster large amount of aligned DNA or SNP sequences. We will cover how to read and write sequence data, how to use the popular programs ms (Hudson 2002) and seq-gen (Rambaut and Grassly 1997) for generating coalescent trees and molecular sequences from within **phyclust**, the main function **phyclust()** for finding population structure, the tree finding program baseml of PAML (Yang 1997, 2007), and Haplo-Clustering (Tzeng 2005). More information about the theory, other package functions, and any changes in future versions can be found on our website Phylogenetic Clustering (Phyloclustering) at https://snoweye.github.io/phyclust/.

Specifically, in Section 2, we introduce the basic data structures of **phyclust** and the I/O functions for reading and writing PHYLIP and FASTA files. In Section 3, we demonstrate how to simulate molecular data using the "ms+seqgen" approach from within R. In Section 4, we briefly describe the phylogenetic clustering method, its implementation in **phyclust()**, the visualization functions, the auxiliary function .EMControl() for choosing the model, initialization method, optimization method, and the EM algorithm variant, and propose a "ms+seqgen+phyclust" approach. In Section 5, we illustrate an extension of Phyloclustering, and propose a "phyclust+paml.baseml" approach. In Section 6, we demonstrate the function haplo.post.prob() for Hap-Clustering.

#### 1.1. Installation and quick start

You can install **phyclust** directly from CRAN at https://cran.r-project.org or by download from our website. On most systems, the easiest installation method is to type the following command into R's terminal:

> install.packages("phyclust")

When it finishes, you can use library() to load the package as

> library("phyclust")

Note that **phyclust** requires the **ape** package (Paradis *et al.* 2004), and **ape** also requires other packages depending on its version. All the required packages will be checked and automatically loaded when the **phyclust** is loading.

You can get started quickly with **phyclust** by using the demo() command in R.

> demo("toy", package = "phyclust")

This demo will produce the three plots shown in Figures 2, 3 and 4, and some of the results reported in the Section 4.3. The demo executes the commands duplicated below. You can find more information about each command in the sections that follow.

```
### Rename the data and obtain true cluster membership from sequence names.
 X <- seq.data.toy$org</pre>
 X.class <- as.numeric(gsub(".*-(.)", "\\1", seq.data.toy$seqname))</pre>
### A dot plot, Figure 2.
 windows()
 plotdots(X, X.class)
### A histogram plot, Figure 3.
 windows()
 plothist(X, X.class)
### A Neighbor-Joining plot, Figure 4.
 ret <- phyclust.edist(X, edist.model = .edist.model[3])</pre>
 ret.tree <- nj(ret)
 windows()
 plotnj(ret.tree, X.class = X.class)
### Fit a EE, JC69 model using emEM initialization, Section 4.3
 EMC.2 <- .EMControl(init.procedure = "emEM")</pre>
 set.seed(1234)
  (ret.2 <- phyclust(X, 4, EMC = EMC.2))</pre>
 RRand(ret.2$class.id, X.class)
```

### 1.2. Getting help

You can look for more examples on the help pages or our website: https://snoweye.github.io/phyclust/. Also, you can email the author at wccsnow@gmail.com. All comments are welcome. Bugs will be fixed and suggestions may be implemented in future versions of phyclust.

# 2. Sequence Data Input and Output

Two types of sequences are supported in **phyclust**, nucleotide and SNP. The supported types are stored in .code.type:

```
> .code.type
[1] "NUCLEOTIDE" "SNP"
```

Phyclust accepts three types of input:

- 1. Data read from a text file in PHYLIP format (Section 2.2).
- 2. Data read from a text file in FASTA format (Section 2.3).
- 3. Data simulated by the ms+seggen approach (Section 3).

The data reading functions read.\*() will return a list object of class seq.data (Section 2.2). Suppose we call the returned list object ret. Then, ret\$org.code and ret\$org are two matrices that store the data. Matrix ret\$org.code contains the original data, e.g. A,G,C,T for nucleotide, and ret\$org contains the data formatted for the computer, e.g. 0,1,2,3 for nucleotide. Matrix ret\$org is translated from ret\$org.code according to the standard encoding (Section 2.1) of the chosen data type and most calculations are done with ret\$org. Phyclust outputs sequence data in two formats: PHYLIP or FASTA (Section 2.4).

### 2.1. Standard coding

Genetic data are represented internally using an integer code, and only the integer values get passed to the C core. The two data frames, .nucleotide and .snp, are used to map between internal integer code (nid or sid) and the human interpretable code (code or code.1).

```
> .nucleotide
  nid code code.l
    0
          Α
          G
2
    1
                   g
          С
3
    2
                   С
4
    3
          Т
                   t
    4
5
> .snp
  sid code
1
    0
          1
2
    1
          2
3
    2
```

Note that we use "-" to indicate gaps and other non general syntax. The methods and functions to deal with gaps are still under development.

#### 2.2. Input PHYLIP format

Some virus data collected from an EIAV-infected pony, #524 (Baccam *et al.* 2003), named "Great pony 524 EIAV rev dataset", are provided as an example of PHYLIP-formatted sequence data. You can view the file with commands

```
> data.path <- paste(.libPaths()[1], "/phyclust/data/pony524.phy", sep = "")
> edit(file = data.path)
```

Below, we show the first 5 sequences and first 50 sites. The first line indicates there are 146 sequences and 405 sites in this file. The sequences are visible starting from the second line, where the first 10 characters are reserved for the sequence name or id.

```
AF314258 gatcctcagg gccctctgga aagtgaccag tggtgcaggg tcctccggca
AF314259 gatcctcagg gccctctgga aagtgaccag tggtgcaggg tcctccggca
AF314260 gatcctcagg gccctctgga aagtgaccag tggtgcaggg tcctccggca
AF314261 gatcctcagg gccctctgga aagtgaccag tggtgcaggg tcctccggca
AF314262 gatcctcagg gccctctgga aagtgaccag tggtgcaggg tcctccggca
AF314262 gatcctcagg gccctctgga aagtgaccag tggtgcaggg tcctccggca
```

By default, function read.phylip() will read in a PHYLIP file and assume the file contains nucleotide sequences. It will read in sequences, translate them using the encoding (Section 2.1), and store them in a list object of class seq.data. The following example reads the Pony 524 dataset.

```
> data.path <- paste(.libPaths()[1], "/phyclust/data/pony524.phy", sep = "")</pre>
> (my.pony.524 <- read.phylip(data.path))</pre>
code.type: NUCLEOTIDE, n.seq: 146, seq.len: 405.
> str(my.pony.524)
List of 7
 $ code.type: chr "NUCLEOTIDE"
         : chr " 146 405"
 $ info
 $ nseq
            : num 146
 $ seqlen : num 405
 $ segname : Named chr [1:146] "AF314258" "AF314259" "AF314260" "AF314261" ...
  ..- attr(*, "names")= chr [1:146] "1" "2" "3" "4" ...
 $ org.code : chr [1:146, 1:405] "g" "g" "g" "g" ...
            : num [1:146, 1:405] 1 1 1 1 1 1 1 1 1 1 ...
 $ org
 - attr(*, "class")= chr "seq.data"
The sample PHYLIP-formatted SNP dataset from a study of Crohn's disease (Hugot et al.
> data.path <- paste(.libPaths()[1], "/phyclust/data/crohn.phy", sep = "")</pre>
```

2001) can be loaded with the commands

```
> (my.snp <- read.phylip(data.path, code.type = .code.type[2]))</pre>
code.type: SNP, n.seq: 1102, seq.len: 8.
```

Notice, the code.type argument must specify the data is of type SNP.

#### 2.3. FASTA format

The sequence data from another pony, #625 (Baccam et al. 2003), named "Great pony 625 EIAV rev dataset", is provided in FASTA format. Here is full-length first sequence in that file. It starts with ">" followed by a sequence id and description on the same line. Subsequent lines contain the actualy sequence until the next line starting with ">".

```
>AF512608 Equine infectious anemia virus isolate R93.3/E98.1 gp45 and rev
GATCCTCAGGGCCCTCTGGAAAGTGACCAGTGGTGCAGGGTCCTTCGGCAGTCACTACCT
GAAGAAAAATTCCATCGCAAACATGCATCGCGAGAAGACACCTGGGACCAGGCCCAACA
CAACATACACCTAGCAGGCGTGACCGGTGGATCAGGGAACAAATACTACAGGCAGAAGTA
CTCCAGGAACGACTGGAATGGAGAATCAGAGGGGTACAACAGGCGGCCAAAGAGCTGGAT
GAAGTCAATCGAGGCATTTGGAGAGAGCTACATTTCCGAGAAGACCAAAAGGGAGATTTC
TCAGCCTGGGGCGGTTATCAACGAGCACAAGAACGGCACTGGGGGGAACAATCCTCACCA
AGGGTCCTTAGACCTGGAGATTCGAAGCGAAGGAGGAAACATTTAT
>AF512609 Equine infectious anemia virus isolate R93.2/E105 ...
```

By default, function read.fasta() will read in a FASTA file and assume the file contains nucleotide sequences. It also returns a list object of class seq.data. The following code example reads the pony #625 dataset.

```
> data.path <- paste(.libPaths()[1], "/phyclust/data/pony625.fas", sep = "")
> (my.pony.625 <- read.fasta.nucleotide(data.path))
code.type: NUCLEOTIDE, n.seq: 62, seq.len: 406.
> str(my.pony.625)
List of 6
$ code.type: chr "NUCLEOTIDE"
$ nseq : num 62
$ seqlen : int 406
$ seqname : chr [1:62] "AF512608" "AF512609" "AF512610" "AF512611" ...
$ org.code : chr [1:62, 1:406] "G" "G" "G" ...
$ org : num [1:62, 1:406] 1 1 1 1 1 1 1 1 1 ...
- attr(*, "class")= chr "seq.data"
```

## 2.4. Saving sequences

To save sequences in a file, you can use the functions write.\*(), which are analogous to the functions read.\*() but take a data matrix X and a file name filename. With the following code, we save the two pony datasets in PHYLIP and FASTA formats to the working directory.

```
> # PHYLIp
> write.phylip(my.pony.625$org, "new.625.txt")
> edit(file = "new.625.txt")
> # FASTA
> write.fasta(my.pony.524$org, "new.524.txt")
> edit(file = "new.524.txt")
```

# 3. The ms+seqgen Approach

Phyclust incorporates two popular open source C programs: ms (Hudson 2002) and seq-gen (Rambaut and Grassly 1997). The original source code and documentation are available on the authors' original websites, or the mirror on phyloclustering website at https://snoweye.github.io/phyclust/.

The ms documentation demonstrates how to use ms to generate coalescent trees, followed by sequence generation using seq-gen, the popular ms+seqgen approach for simulating molecular data. I have edited the source code slightly to make these commands available through new R functions ms() and seqgen(). This solution eases the burden on the user, bypassing the need to compile both programs. Moreover, combining these functions with the phyclust() function produces a ms+seqgen+phyclust approach for simulation and bootstrap studies (see Section 4.4).

#### 3.1. Using the ms() function to generate trees

Almost all command line options of program **ms** are available through option **opts** in **ms()**. Call the function without arguments to see all the options.

```
> ms()
> ?ms
```

The following example generates a coalescent tree (ms option -T) with 3 leaves (nsam = 3) and population growth rate 0.1 (-G 0.1). Function ms() returns ms text output stored in an array, one line per element. The third line contains the tree in NEWICK format, which can be read by the read.tree() function in the ape package (Paradis et al. 2004). Function read.tree() returns an object of class phylo, which can be drawn by function plot() or plot.phylo() of the ape package.

```
> set.seed(1234)
> (ret.ms <- ms(nsam = 3, opts = "-T -G 0.1"))
ms 3 1 -T -G 0.1
//
(s1: 0.568774938583,(s2: 0.355949461460,s3: 0.355949461460): 0.212825477123);
> (tree.anc <- read.tree(text = ret.ms[3]))

Phylogenetic tree with 3 tips and 2 internal nodes.

Tip labels:
[1] "s1" "s2" "s3"

Rooted; includes branch lengths.
> tree.anc$tip.label <- paste("a", 1:3, sep = "")
> plot(tree.anc, type = "c")
> axisPhylo()
```

## 3.2. Using the seqgen() function to generate sequences

Almost all options of the command line program **seq-gen** are available from within R via the option **opts** in the **seqgen()** function. Call the function without arguments to see the options.

```
> seqgen()
> ?seqgen
```

The seqgen() function requires a rooted tree in NEWICK format or an object of class phylo. In the following, we demonstrate the ms+seqgen approach to generate sequences. The result is a character vector of class seqgen, which contains 5 sequences, each of 40 bases (seq-gen option -140). The option -mHKY specifies the HKY85 model of evolution (Hasegawa et al. 1985). Without further options, HKY85 is equivalent to the JC69 model (Jukes and Cantor 1969).

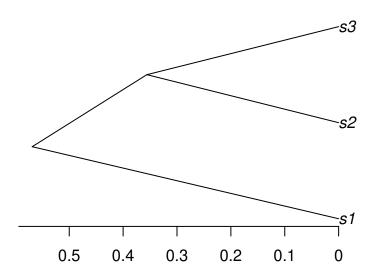


Figure 1: A diagram of a simple coalescent tree.

```
s5
          CTCTCTCTGGACGCACACTTTAAGGGGGGGATTGAACTACA
s2
          CTCTTCGGGCTCGGATAAGTTTGGAGGGTTGTTCTCTACA
          CTCTGAGTGCTCGGATTAGTTAGGGGGGAATGACGTCTACA
s3
          CTCTTATCTCTCGGATAAGTTGGGGGTGATGGCTTTTACA
s4
> set.seed(123)
> (ret.seq <- seqgen(opts = "-mHKY -140 -q", rooted.tree = tree.anc))</pre>
5 40
          CTCTCATTGGACGCACACTTTAGGGGGGGGATTGCACTGCA
s1
s5
          CTCTCTCTGGACGCACACTTTAAGGGGGGGATTGAACTACA
          CTCTTCGGGCTCGGATAAGTTTGGAGGGTTGTTCTCTACA
s2
s3
          \tt CTCTGAGTGCTCGGATTAGTTAGGGGGGAATGACGTCTACA
          CTCTTATCTCTCGGATAAGTTGGGGGTGATGGCTTTTACA
s4
> str(ret.seq)
Class 'seqgen'
                chr [1:6] " 5 40" "s1
                                               CTCTCATTGGACGCACACTTTAGGGGGG ...
```

The seqgen() function need not take in a tree from ms(), but ms() provides options to construct trees of different shapes using coalescent theory. Also, you can provide seqgen() with an ancestral sequence that is then evolved along the given tree (see Section 3.3).

### 3.3. Inputing an ancestral sequence to ms+seqgen

Phyclust provides two functions gen.seq.HKY() and gen.seq.SNP() to implement the ms+seqgen

approach under wide-ranging parameter choices. A rooted tree is required and an ancestral sequence is an option.

The following example generates a tree and provides an ancestral sequence. Function seqgen() will use parameters  $\kappa$  (kappa) and  $\pi_A, \pi_G, \pi_C, \pi_T$  (pi.HKY) to evolve the ancestral sequence (anc.HKY) down the tree. The function read.seqgen() reads the seqgen() object and returns a new dataset of class seq.data for use by the function phyclust() (see Section 4.2).

```
> # Generate a tree
> set.seed(1234)
> ret.ms <- ms(nsam = 5, nreps = 1, opts = "-T")
> tree.ms <- read.tree(text = ret.ms[3])</pre>
> # Generate nucleotide sequences
> (anc.HKY < - rep(0:3, 3))
 [1] 0 1 2 3 0 1 2 3 0 1 2 3
> paste(nid2code(anc.HKY, lower.case = FALSE), collapse = "")
[1] "AGCTAGCTAGCT"
> pi.HKY <- c(0.2, 0.2, 0.3, 0.3)
> kappa <- 1.1
> L <- length(anc.HKY)
> set.seed(1234)
> (HKY.1 <- gen.seq.HKY(tree.ms, pi.HKY, kappa, L, anc.seq = anc.HKY))</pre>
 5 12
s1
          AGCTTGACCGGC
s3
          AGCTTCACCGGT
s2
          ACCTCGCTAGCT
          ACGACGCTCGCT
s4
          CCTACGCTAGCT
s5
> (ret <- read.seggen(HKY.1))</pre>
code.type: NUCLEOTIDE, n.seq: 5, seq.len: 12.
```

Function gen.seq.HKY() may be a good example for advanced users wanting to simulate more complex evolutionary processes, such as recombination, migration and island models. It passes an option input to seqgen(), which in this case is used to pass the ancestral sequence, but could be used to pass other options available in the seq-gen program. The option input takes in a character vector (including the tree) where each element contains one seq-gen option. Function seqgen() writes these options to a temporary file, which is later communicated to seq-gen.

```
" -u", ttips + 1, " -k1", " -q", sep = "")
ret <- seqgen(opts, input = input)

### Partial source code of seqgen().
    if (!is.null(newick.tree)) {
        write(newick.tree, file = temp.file.ms, sep = "")
    }
    else if (!is.null(input)) {
        write(input, file = temp.file.ms, sep = "\n")
    }
    else {
        stop("A newick or rooted/phylo tree is required.")
}</pre>
```

# 4. Phylogenetic Clustering (Phyloclustering)

Phylogenetic clustering (phyloclustering) is an evolutionary Continuous Time Markov Chain (CTMC) model-based approach to identify population structure from molecular data without assuming linkage equilibrium. Let  $X = (x_{nl})_{N \times L}$  be the data matrix containing N sequences observed at L sites. Denote the molecular sequence of individual n as  $x_n = (x_{n1}, \dots, x_{nL}) \in \mathfrak{X}$ and  $x_{nl} \in \mathcal{S}$  where  $\mathfrak{X}$  contains all possible sequences of length L from alphabet  $\mathcal{S}$ , e.g.  $\mathcal{S} =$ {A, G, C, T} for nucleotide sequences. A finite mixture model provides a statistical framework for clustering. In this setting, each individual sequence  $x_n$  is independent and identically drawn from  $f(\boldsymbol{x}_n|\boldsymbol{\eta},\boldsymbol{\Theta}) = \sum_{k=1}^K \eta_k f_k(\boldsymbol{x}_n|\boldsymbol{\Theta}_k)$  where  $f_k()$  is the density for the kth component,  $\eta = \{\eta_1, \dots, \eta_K\}$  are the mixing proportions summing to one, and  $\Theta = \{\Theta_1, \dots, \Theta_K\}$  contains parameters for the components (Fraley and Raftery 2002). Component  $f_k()$  is modeled as a transition probability  $p_{\mu_k,x_n}(t_k)$  from a CTMC mutation process (Felsenstein 2004), where a sequence  $x_n$  evolves from an ancestor  $\mu_k = (\mu_{k1}, \dots, \mu_{kL}) \in \mathfrak{X}$  representing the kth cluster. The evolutionary process is modeled with instantaneous rate matrix  $Q_k$  and time  $t_k$  which are allowed to differ by cluster, so that  $\Theta_k = \{\mu_k, Q_k, t_k\}$ . The likelihood is maximized by an EM algorithm (Dempster et al. 1977), sequences are classified by the maximum posterior probabilities, and the number of clusters is assessed by bootstrap (Maitra and Melnykov 2010).

Available choices for the  $Q_k$  parameterization in **phyclust** include JC69 (Jukes and Cantor 1969), K80 (Kimura 1980), and HKY85 (Hasegawa *et al.* 1985). These choices are listed in .substitution.model. In addition,  $Q_k$  and  $t_k$  can be constrained across clusters as shown in Table 1 (also see .identifier).

The available initialization methods (.init.method) for the EM algorithm use pairwise distances, and the available models for computing the evolutionary distance are listed in .edist.model. The model used for computing distances need not match the model used to model evolution in phylogenetic clustering (in .substitution.model). There are additional pairwise distance models available in the ape package (Paradis et al. 2004).

The .show.option() function lists all options available in the **phyclust** package. These options can be used in the .EMControl() function to generate an options object (such as .EMC), which defines the selected phyloclust model, the initialization method, the EM algorithm and

Table 1: Combinations of Models			
Identifier	Q	t	
EE	$Q_1 = Q_2 = \cdots = Q_K$	$t_1 = t_2 = \dots = t_K$	
EV	$Q_1 = Q_2 = \cdots = Q_K$	$t_1 \neq t_2 \neq \cdots \neq t_K$	
VE	$oldsymbol{Q}_1  eq oldsymbol{Q}_2 \cdots  eq oldsymbol{Q}_K$	$t_1 = t_2 = \dots = t_K$	
VV	$oldsymbol{Q}_1  eq oldsymbol{Q}_2  eq \cdots  eq oldsymbol{Q}_K$	$t_1 \neq t_2 \neq \cdots \neq t_K$	

the data type. This argument is passed to function phyclust() using argument EMC. All options are explained in the help pages. The best choices for options may vary with application. In particular, initialization can be tricky, and you should try several initialization algorithms (see Section 4.3).

```
> .show.option()
boundary method: ADJUST, IGNORE
code type: NUCLEOTIDE, SNP
edist model: D_JC69, D_K80, D_HAMMING
em method: EM, ECM, AECM
identifier: EE, EV, VE, VV
init method: randomMu, NJ, randomNJ, PAM, K-Medoids, manualMu
init procedure: exhaustEM, emEM, RndEM, RndpEM
standard code:
     nid code code.l
[1,]
       0
            Α
[2,]
       1
            G
                    g
[3,]
       2
            С
                    С
       3
            Τ
[4,]
                    t
[5,]
       4
     sid code
[1,]
       0
            1
[2,]
       1
[3,]
       2
substitution model:
          model code.type
 [1,]
           JC69 NUCLEOTIDE
 [2,]
            K80 NUCLEOTIDE
 [3,]
            F81 NUCLEOTIDE
 [4,]
          HKY85 NUCLEOTIDE
 [5,]
       SNP_JC69
                        SNP
        SNP_F81
 [6,]
                        SNP
 [7,]
          E_F81 NUCLEOTIDE
 [8,]
        E_HKY85 NUCLEOTIDE
 [9,] E_SNP_F81
                        SNP
```

### 4.1. Exploring data

**Phyclust** has functions to help visualize large datasets. We have prepared a simulated dataset (seq.data.toy) with 100 nucleotide sequences of length 200 sites from 4 clusters. The ancestral sequences were simulated using the HKY85 model (Hasegawa et al. 1985) along a tree of height 0.15 (expected number of mutations per site). The observed sequences were simulated along independent trees with height 0.09 descending from the ancestors.

We use X to indicate the data and X.class to indicate the classification which can be a result (class.id) of the phyclust() function or known, in the case of simulation (ms+seqgen). The ms+seq-gen simulation procedure within phyclust produces sequences with names "sequence.id-class.id," so the R function gsub() (for details, type ?gsub) can be used to extract the sequence classification (class id). In this section, we demonstrate visualization of a simulated dataset, where the clusters are know. Similar figures can be produced for real datasets with clusters estimated by phyclust().

The following code produces the plot of Figure 2. Each row represents a sequence and each column represents a site. By default, it will show all changes with respect to the consensus sequence. If the X.class is omitted, then it draws in the original order of data, otherwise in the cluster order.

```
> seq.data.toy
code.type: NUCLEOTIDE, n.seq: 100, seq.len: 200.
> X <- seq.data.toy$org
> X.class <- as.numeric(gsub(".*-(.)", "\\1", seq.data.toy$seqname))
> plotdots(X, X.class)
```

The chosen sequence is fully colored, with green, blue, purple and red representing nucleotides A, G, C, and T. All other sequences show only mutant sites compared to the consensus sequence. The dashed lines separate the clusters. The bottom row indicates the segregating sites, i.e. those sites containing at least one mutation. By default *only* segregating sites are shown. Type ?plotdots for more information.

Next, we may wish to see how many mutations each sequence has relative to a reference sequence. The following code prepares the plot of Figure 3, showing the number of mutations of all sequences within each cluster relative to the chosen reference sequence. The top plot is for the whole dataset. The other plots are for the four clusters.

```
> plothist(X, X.class)
```

Last, we may like to visualize clusters on a more traditional diagram of evolutionary relationships, the phylogenetic tree. The following code produces Figure 4. The phyclust.edist() function takes in a data matrix X, computes and returns pairwise distances for all sequences using the Hamming distance as a distance measure (.edist.model[3] is D\_HAMMING). The neighbor-joining method (Saitou and Nei 1987) is used to build a tree from the distance matrix. The function plotnj() is a function in phyclust for plotting the resulting tree with branches colored according to the clusters defined in argument X.class. These clusters may be provided by the user (as is the case here) or as a result of inferring the clusters using phyclust().

```
> (ret <- phyclust.edist(X, edist.model = .edist.model[3]))
Class 'dist' atomic [1:4950] 4 3 4 7 2 4 5 5 8 2 ...</pre>
```

# **Dots Plot**

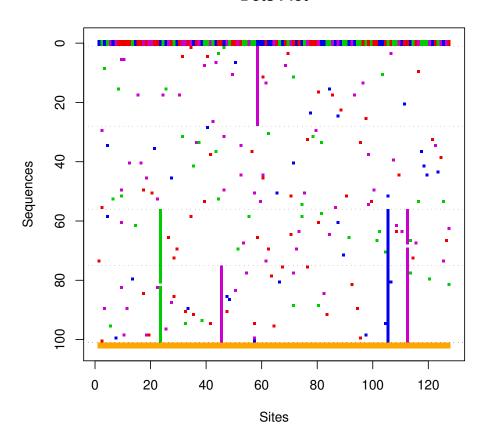


Figure 2: A dot plot for the toy dataset.

```
..- attr(*, "Size")= int 100
..- attr(*, "Diag")= logi FALSE
..- attr(*, "Upper")= logi FALSE
..- attr(*, "method")= chr "D_HAMMING"
> (ret.tree <- nj(ret))</pre>
```

Phylogenetic tree with 100 tips and 98 internal nodes.

```
Tip labels:
1, 2, 3, 4, 5, 6, ...
```

Unrooted; includes branch lengths.
> plotnj(ret.tree, X.class = X.class)

# 4.2. Using the phyclust() function

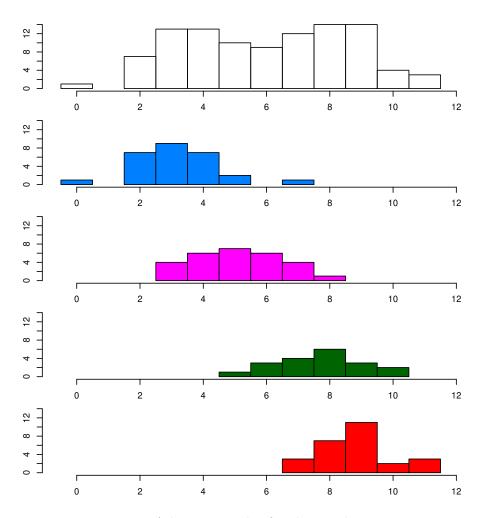


Figure 3: A histogram plot for the toy dataset.

We will use the toy dataset to demonstrate the phyclust() function, which requires two arguments: the data matrix X and the number of clusters K. The optional EMC argument of phyclust() is used to pass in model and optimization choices. By default, the object .EMC is passed to phyclust(). See Section 4.3 for more information about changing the defaults. In the following example, we use the defaults to fit 4 clusters to the toy data.

```
> set.seed(1234)
> (ret.1 <- phyclust(X, 4))
Phyclust Results:
code type: NUCLEOTIDE, em method: EM, boundary method: ADJUST.
init procedure: exhaustEM, method: randomMu.
model substitution: JC69, distance: D_JC69.
iter: 37 3158 0, convergence: 0, check.param: 1.
eps: 4.851e-13, error: 0.
N.X.org: 100, N.X.unique: 87, L: 200, K: 4, p: 804, N.seg.site: 127.
logL: -1439, bic: 6581, aic: 4487, icl: 6588</pre>
```

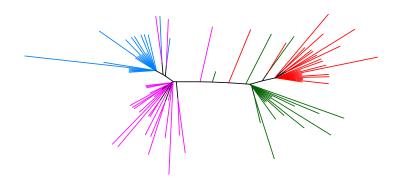


Figure 4: A NJ tree for the toy dataset.

```
identifier: EE
  Eta: 0.4360 0.01149 0.284 0.2700
  Tt: 0.003325
  n.class: 44 1 28 27
> RRand(ret.1$class.id, X.class)
   Rand adjRand Eindex
  0.9018 0.7653 0.1655
> class(ret.1)
[1] "phyclust"
```

The output of the call to phyclust() includes

```
N.X.org
                   the number of sequences
     N.X.unique
                   the number of unique/distinct sequences
                   for the number of sites
     N.seq.site
                   for the number of segregating sites
                   the number of clusters
                   the number of parameters
                  the maximum likelihood value
            logL
                  for BIC, AIC and ICL
bic, aic and icl
                   the model choice for Q_k and t_k (see Table 1)
     identifier
             Eta
                   mixing proportions \eta
              Tt
```

A quick glance at the results shows that the default settings did not produce good results. There is a degenerate cluster (only one member), as indicated by the count of members in each of the four classes: n.class. The default initialization procedure is exhaustEM and the default initialization method is randomMu, which means it randomly picks 4 sequences to be the cluster centers and runs the EM algorithm to convergence. While the EM algorithm is guaranteed to converge, it may only find a local optimum. It is important to try multiple random initializations to improve your chances of finding the global maximum. The adjusted Rand index (Hubert and Arabie 1985), adjRand, which can be used to compare two clusterings, is about 0.7653 when comparing the phyclust() solution to the true clusters. It should be 1.000 for perfect agreement.

The phyclust() function returns a list object of class phyclust. You can type str(ret.1) to get the details of the phyclust class, or type ?phyclust to see descriptions of all elements. Section 4.3 will give more information for controlling the phyclust() function. This object can also be used as input to other functions, for example bootstrapping and simulations (see Section 4.4).

#### 4.3. Using the .EMControl() function

The .EMControl() function provides a list object that can be used as argument EMC to phyclust(). With no arguments, it returns the default values. The internal object .EMC is a template object holding the defaults. Each element configures some aspect of the evolutionary model, phyloclust model, initialization, optimization, and EM algorithm. See the help page for details and visit our website for examples.

```
> ?.EMControl
> ?.EMC
```

You can either modify the template .EMC directly, or use the function .EMControl() to generate a new control object. The following example modifies an object copied from the template. It changes the initialization method to "emEM," which yields a better solution than our previous attempt (ret.1). The adjusted Rand index is now 1.000, indicating a perfect match between the truth and inferred structures.

```
> EMC.2 <- .EMC
> EMC.2$init.procedure <- .init.procedure[2]
> ### The same as
> ### EMC.2 <- .EMControl(init.procedure = "emEM")
> set.seed(1234)
> (ret.2 \leftarrow phyclust(X, 4, EMC = EMC.2))
Phyclust Results:
code type: NUCLEOTIDE, em method: EM, boundary method: ADJUST.
init procedure: emEM, method: randomMu.
model substitution: JC69, distance: D_JC69.
iter: 103 8725 0, convergence: 0, check.param: 1.
eps: 2.753e-14, error: 0.
N.X.org: 100, N.X.unique: 87, L: 200, K: 4, p: 804, N.seg.site: 127.
logL: -1379, bic: 6461, aic: 4367, icl: 6469
identifier: EE
  Eta: 0.2700 0.1898 0.2801 0.2602
```

```
Tt: 0.003074
n.class: 27 19 28 26
> RRand(ret.2$class.id, X.class)
    Rand adjRand Eindex
1.0000 1.0000 0.1209
```

Now, we use the function .EMControl() to generate a new control that uses "RndEM" for initialization. It also changes the phyloclustering model to use the "EV" variant (see Table 1). The data was simulated under "EE" conditions, so this is an over-parameterized model. Such models may also tend to infer degenerate clusters, and again, more initializations may be required for good results. From the output, we observe the mixing proprtion Eta of the second cluster is smaller than others. In addition, the evolutionary time Tt of this cluster is unusually larger compared to the others. These are both indications of a degenerate cluster, and indeed, no sequences were assigned to this cluster.

```
> EMC.3 <- .EMControl(init.procedure = "RndEM", identifier = "EV")
> ### The same as
> ### EMC.3 <- .EMC
> ### EMC.3$init.procedure <- .init.procedure[3]
> ### EMC.3$identifer <- .identifier[3]
> set.seed(1234)
> (ret.3 \leftarrow phyclust(X, 4, EMC = EMC.3))
Phyclust Results:
code type: NUCLEOTIDE, em method: EM, boundary method: ADJUST.
init procedure: RndEM, method: randomMu.
model substitution: JC69, distance: D_JC69.
iter: 104 51836 0, convergence: 0, check.param: 1.
eps: 4.278e-13, error: 0.
N.X.org: 100, N.X.unique: 87, L: 200, K: 4, p: 807, N.seg.site: 127.
logL: -1453, bic: 6621, aic: 4519, icl: 6627
identifier: EV
  Eta: 0.2696 0.01149 0.2844 0.4461
  Tt: 0.002230 4.75 0.003663 0.003924
  n.class: 27 0 28 45
> RRand(ret.3$class.id, X.class)
   Rand adjRand Eindex
 0.9002 0.7640 0.1698
```

There is a convenient function find.best() (Section 4.4) that is useful for finding the highest likelihood fit among multiple calls to phyclust() with varying arguments. This function runs phyclust() repeatedly on combinations of selected initialization options by updating an internal EMC control object in each iteration. Please be patient, as this function may take some time to complete.

### 4.4. The ms+seqgen+phyclust approach

Model selection includes identifying the number of clusters, type of evolutionary model, and phyloclustering assumptions (Table 1). We could use information criteria to choose among

models, but the parameter space is mixed continuous and discrete so the theory justifying these critiera does not apply. A more elaborate procedure to assess the number of clusters for a dataset is based on the parametric bootstrap technique and sequential hypothesis testing (Maitra and Melnykov 2010).

The basic idea is to resample datasets from the fitted model using the functions ms() and seqgen(), and refit the resampled dataset by phyclust(). The same fitting method is applied to each dataset, producing a distribution of parameter estimates. The bootstrap.seq.data() function is a tool for this procedure; it takes a fitted model, the output of a previous call to phyclust(). The following example bootstraps the toy dataset once assuming K=2 clusters.

```
> set.seed(1234)
> ret.4 <- phyclust(X, 2)</pre>
> (seq.data.toy.new <- bootstrap.seq.data(ret.4))</pre>
code.type: NUCLEOTIDE, n.seq: 100, seq.len: 200.
> (ret.4.new <- phyclust(seq.data.toy.new$org, 2))</pre>
Phyclust Results:
code type: NUCLEOTIDE, em method: EM, boundary method: ADJUST.
init procedure: exhaustEM, method: randomMu.
model substitution: JC69, distance: D_JC69.
iter: 30 2947 0, convergence: 0, check.param: 1.
eps: 1.41e-14, error: 0.
N.X.org: 100, N.X.unique: 56, L: 200, K: 2, p: 402, N.seg.site: 69.
logL: -685.3, bic: 3222, aic: 2175, icl: 3222
identifier: EE
  Eta: 0.48 0.52
  Tt: 0.001354
  n.class: 48 52
```

Output ret.4 is the result from phyclust(), seq.data.toy.new is a new dataset bootstrapped from the model in ret.4, and ret.4.new is the fit of the bootstrapped dataset. Generally, we need to repeat these steps several times to obtain a distribution of parameter estimates. For example, the following code uses B=10 bootstraps to obtain the distributions of log-likelihood for K=1 and K=2, and we can perform a likelihood ratio test to assess the number of clusters.

```
> ### This code may take a long time to run.
> set.seed(1234)
> ret.K1 <- find.best(X, 1)
> ret.K2 <- find.best(X, 2)
> (logL.diff <- ret.K2$logL - ret.K1$logL)
[1] 663.2635
> set.seed(1234)
> B <- 10
> boot.logL.diff <- NULL
> for(b in 1:B){
> seq.data.toy.new <- bootstrap.seq.data(ret.K1)</pre>
```

```
> ret.H0 <- find.best(seq.data.toy.new$org, 1)
> ret.H1.H0 <- find.best(seq.data.toy.new$org, 2)
> boot.logL.diff <- c(boot.logL.diff, ret.H1.H0$logL - ret.H0$logL)
> }
> boot.logL.diff
[1] 20.17399 24.94449 25.50644 24.30299 25.94984 19.03801 19.20099 19.40565
[9] 18.88133 37.16275
> sum(logL.diff < boot.logL.diff)
[1] 0</pre>
```

Here, the difference of likelihood values logL.idff is 663.2635 between models K=1 and K=2. We bootstrap from a fitted model of K=1, and it is 0 out of 10 results indicating the K=2 performs better than K=1. As the B increases, we may more confident to reject K=1.

# 5. Phylogenetic Analysis by Maximum Likelihood (PAML)

PAML (Yang 1997, 2007) is one of the popular packages for estimating phylogenetic trees given nucleotide, codon, or amino acid sequences. It searches the optimal tree by maximum likelihood, and implements several different options, models, algorithms and statistical tests to finding appropriate trees which can interpreter the relations of sequences or species in an evolutionary manner. The package contains several programs to perform the phylogenetic analysis. The original source code and documentation are available on the authors' original website, or the mirror on phyloclustering website at https://snoweye.github.io/phyclust/.

Among the programs, only baseml is ported into phyclust, and the main function paml.baseml() in R can find trees for nucleotide sequences. Most of functionalities of baseml are transfered correctly, but I disable some advance options due to complexity of input and output. I also provide options for paml.baseml() to bridge the options of ms(), seqgen(), and phyclust(). The following Section 5.1 introduces the usage of paml.baseml() and it's control function. Section 5.2 illustrates examples for finding the ancestral tree given central sequences of clusters. This provides a two-stages approach to building a phylogeny on a large dataset, and is

#### 5.1. Using the paml.baseml() function

The inputs of baseml relies on several files including the tree file, sequence file, and control file. The outputs of baseml are also several files depending on the controls. The main function paml.baseml() is implemented in the same way of ms() and seqgen(). The I/O design is to all store to and read from a temporary directory which will be removed after computing is finish. Use the following code to see the default controls and the details of the main function.

```
> ?paml.baseml
> paml.baseml.show.default()
```

an extension of Phyloclustering.

The following example uses paml.baseml() to obtain the best tree for the given model and to compare the best to the true tree. Here, we only use a small data (five sequences) to test the program. Note that the best is in terms of the highest likelihood of the given model among

a finite subset of tree space. This tree may or may not a good candidate to interpreter the data. For a large dataset, it may take long time to find the best tree.

```
> ### Generate data.
> set.seed(123)
> ret.ms <- ms(nsam = 5, nreps = 1, opts = "-T")
> ret.seggen <- seggen(opts = "-mHKY -140 -s0.2", newick.tree = ret.ms[3])
> (ret.nucleotide <- read.seqgen(ret.seqgen))</pre>
code.type: NUCLEOTIDE, n.seq: 5, seq.len: 40, byrow: TRUE.
> X <- ret.nucleotide$org
> seqname <- ret.nucleotide$seqname
> ### Run baseml.
> opts <- paml.baseml.control(model = 4, clock = 1)</pre>
> (ret.baseml <- paml.baseml(X, seqname = seqname, opts = opts))</pre>
((s1: 0.000000, s5: 0.000000): 0.091018, ((s2: 0.012639, s4: 0.012639): 0.02...
> (ret.baseml.init <- paml.baseml(X, seqname = seqname, opts = opts,</pre>
     newick.trees = ret.ms[3]))
((s1: 0.000000, s5: 0.000000): 0.091018, ((s2: 0.012639, s4: 0.012639): 0.02...
> ret.ms[3]
[1] "((s1: 0.072680674493,s5: 0.072680674493): 0.498367525637,(s2: 0.1790905...
> ### Unrooted tree.
> opts <- paml.baseml.control(model = 4)</pre>
> (ret.baseml.unrooted <- paml.baseml(X, seqname = seqname, opts = opts))</pre>
(((s1: 0.000004, s5: 0.000004): 0.141411, s2: 0.000004): 0.025586, s3: 0.052...
> par(mfrow = c(2, 2))
> plot(read.tree(text = ret.ms[3]), main = "true")
> plot(read.tree(text = ret.baseml$best.tree), main = "baseml")
> plot(read.tree(text = ret.baseml.init$best.tree),
       main = "baseml with initial")
> plot(unroot(read.tree(text = ret.baseml.unrooted$best.tree)),
       main = "baseml unrooted")
```

The option opts of paml.baseml() will take a list generated by paml.baseml.control() by default. The output of paml.baseml() is also a list containing all files as dumped by PAML. The element mlb contains the major results as the option outfile specified by PAML. The element stdout contains all STDOUT which is directly printed by PAML and is controlled by the options noisy and verbose. Note that due to the complexity of input and output, the multiple genes option may be disable.

Warning: The ms() generates an ultimate tree which is a rooted tree with the same height to all leaves, but the PAML will search an unrooted tree by default. PAML may also return a rooted tree for an unrooted tree with multifurcation at the root.

### 5.2. The phyclust+paml.baseml approach

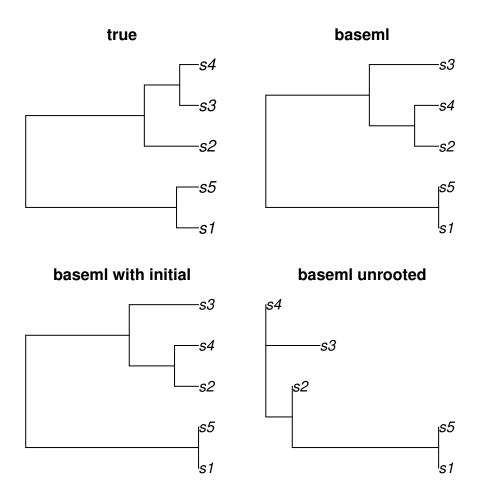


Figure 5: A true tree and an estimation.

For a large dataset, it is a straight forward to consider the combination of the phyclust() and paml.baseml() where Phyloclustering find central sequences for populations and PAML find the phylogeny for the central sequences. Similary to the supertree (Bininda-Emonds 2004; Bininda-Emonds et al. 2002), this approach provides a different way of finding phylogeny to the phylogenetic analysis. Note that the best number of clusters and the best model for the phylogeny are all needed to be determined, either by information criteria or by bootstrap.

The following is an interesting illustration for the toy dataset in **phyclust** using **phyclust()** followed by **paml.baseml()**. The Figure 4 is the result for the same dataset comparing to this Figure 6, while the former uses the distance approach and the later uses the maximum likelihood approachs (Phyloclustering and PAML). Note that the Figure 6 has the same topology as the true where the toy data genetated from.

```
> ### Fit a EE, JC69 model using emEM and fit an ancestral tree.
> EMC <- .EMControl(init.procedure = "emEM")
> set.seed(1234)
```

```
> ret.K <- phyclust(seq.data.toy$org, K, EMC = EMC)
> (ret.Mu <- paml.baseml(ret.K$Mu, opts = paml.baseml.control()))
> tree.est <- read.tree(text = ret.Mu$best.tree)
>
> ### Construct descent trees.
> for(k in 1:K){
> tree.dec <- gen.star.tree(ret.K$n.class[k], total.height = ret.K$QA$Tt)
> tree.est <- bind.tree(tree.est, tree.dec,
> where = which(tree.est$tip.label == as.character(k)))
> }
> est.class <- rep(1:K, ret.K$n.class)
> plotnj(unroot(tree.est), X.class = est.class, main = "tree of sequences")
```

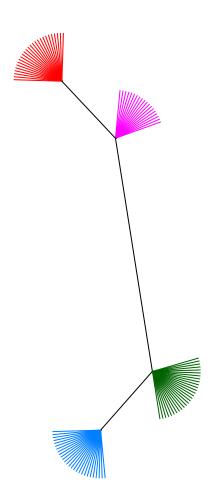


Figure 6: An estimation tree from phyclust() and paml.baseml().

# 6. Using the haplo.post.prob() function for Hap-Clustering

Haplotype grouping (Tzeng 2005) for SNP datasets can be viewed as an alternative method to phyloclustering. The author's R code has been integrate into **phyclust**, and the original function has been renamed **haplo.post.prob()**. The example used by the author is the Crohn's disease dataset (Hugot *et al.* 2001), which is also included in the **phyclust** package. The original description of the author's R code is in the install directory **phyclust/doc/Documents/tzeng-Readme.txt** or in the source code directory **phyclust/inst/doc/Documents/tzeng-Readme.txt**.

The following example returns the same results as Tzeng (2005), where the predicted number of clusters based on her information criterion is 13. The function returns a list object, here stored in ret. The list element ret\$haplo stores information about the SNP sequences, ret\$FD.id and ret\$RD.id store the full and reduced dimensional indices, ret\$FD.post and ret\$RD.post store the full and reduced dimensional posterior probabilities, and g.truncate shows the number of clusters, the truncated results as described in Tzeng (2005).

```
> data.path <- paste(.libPaths()[1], "/phyclust/data/crohn.phy", sep = "")</pre>
> my.snp <- read.phylip(data.path, code.type = "SNP")</pre>
> ret <- haplo.post.prob(my.snp$org, ploidy = 1)</pre>
> str(ret)
List of 6
 $ haplo
             :List of 6
  ..$ haplotype: num [1:39, 1:8] 0 1 1 0 1 1 0 1 1 0 ...
  ..$ hap.prob : num [1:39] 0.00454 0.00181 0.11797 0.00635 0.00635 ...
               : num [1:1102] 1 1 1 1 1 1 1 1 1 1 ...
  ..$ hap1code : int [1:1102] 1 1 1 1 1 2 2 3 3 3 ...
  ..$ hap2code : int [1:1102] 1 1 1 1 1 2 2 3 3 3 ...
  ..$ indx.subj: int [1:1102] 1 2 3 4 5 6 7 8 9 10 ...
 $ FD.id
             : int [1:39] 3 9 18 22 27 28 30 31 34 35 ...
 $ RD.id
             : int [1:13] 3 9 18 22 27 28 30 31 34 35 ...
             : num [1:1102, 1:39] 0 0 0 0 0 0 0 1 1 1 ...
 $ FD.post
             : num [1:1102, 1:13] 0 0 0 0 0 1 1 1 1 1 ...
 $ RD.post
 $ g.truncate: int 13
> getcut.fun(sort(ret$haplo$hap.prob, decreasing = TRUE),
             nn = my.snp$nseq, plot = 1)
```

The getcut.fun() produces a plot based on the information criterion, which can be used to visualize the truncated dimension. In Figure 7, the horizontal line indicates the cut point of 13 haplotypes.

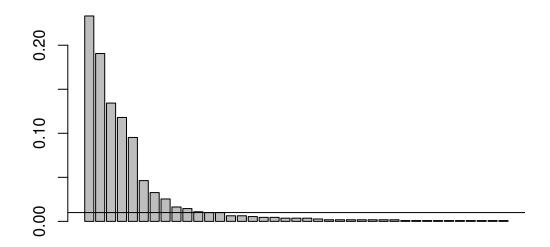


Figure 7: A getcut plot for the Crohn's disease dataset.

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