dist.GDM(clusterSim)

The General Distance Measure
(see Walesiak [2006])

The General Distance Measure (*GDM*) is given by the following equation:

$$GDM = \frac{1}{2} - \frac{\sum_{j=1}^{m} w_j a_{ikj} b_{kij} + \sum_{j=1}^{m} \sum_{\substack{l=1 \\ l \neq i,k}}^{n} w_j a_{ilj} b_{klj}}{2\left[\sum_{j=1}^{m} \sum_{l=1}^{n} w_j a_{ilj}^2 \cdot \sum_{j=1}^{m} \sum_{l=1}^{n} w_j b_{klj}^2\right]^{\frac{1}{2}}}, \tag{1}$$

where: $i, k, l = 1, \ldots, n$ – the number of object,
$j = 1, \ldots, m$ – the number of variable,
x_{ij} (x_{kj}, x_{lj}) – *i*-th (*k*-th, *l*-th) observation on *j*-th variable,
w_j – weight of *j*-th variable: $w_j \in [0;\ 1]$ and $\sum_{j=1}^{m} w_j = 1$ or $w_j \in [0;\ m]$ and $\sum_{j=1}^{m} w_j = m$.

For the variables measured on **ratio and (or) interval scale** we take a_{ipj}, b_{krj} given as (Walesiak [2002]):

$$\begin{aligned} a_{ipj} &= x_{ij} - x_{pj} \quad \text{for} \quad p = k, l \\ b_{krj} &= x_{kj} - x_{rj} \quad \text{for} \quad r = i, l' \end{aligned} \tag{2}$$

$$GDM1 = \frac{1}{2} - \frac{\sum_{j=1}^{m} w_j (x_{ij} - x_{kj})(x_{kj} - x_{ij}) + \sum_{j=1}^{m} \sum_{\substack{l=1 \\ l \neq i,k}}^{n} w_j (x_{ij} - x_{lj})(x_{kj} - x_{lj})}{2\left[\sum_{j=1}^{m} \sum_{l=1}^{n} w_j (x_{ij} - x_{lj})^2 \cdot \sum_{j=1}^{m} \sum_{l=1}^{n} w_j (x_{kj} - x_{lj})^2\right]^{\frac{1}{2}}}. \tag{3}$$

For the variables measured on **ordinal scale** we take a_{ipj}, b_{krj} given as (Walesiak [1993], pp. 44-45):

$$a_{ipj}(b_{krj}) = \begin{cases} 1 & \text{if} \quad x_{ij} > x_{pj}(x_{kj} > x_{rj}) \\ 0 & \text{if} \quad x_{ij} = x_{pj}(x_{kj} = x_{rj}) \\ -1 & \text{if} \quad x_{ij} < x_{pj}(x_{kj} < x_{rj}) \end{cases} \text{for } p = k, l;\ l, r = i, l, \tag{4}$$

$$GDM2 = \frac{1}{2} - \frac{\sum_{j=1}^{m} w_j a_{ikj} b_{kij} + \sum_{j=1}^{m} \sum_{\substack{l=1 \\ l \neq i,k}}^{n} w_j a_{ilj} b_{klj}}{2\left[\sum_{j=1}^{m} \sum_{l=1}^{n} w_j a_{ilj}^2 \cdot \sum_{j=1}^{m} \sum_{l=1}^{n} w_j b_{klj}^2\right]^{\frac{1}{2}}}. \tag{5}$$

The properties of the General Distance Measure

– it can be applied when the variables are measured on the ordinal (*GDM*2), interval and ratio scale (*GDM*1),

– it takes values from the [0; 1] interval. Value 0 indicates that for the compared objects *i, k* between corresponding observations of variables, only relations "equal to" take place. For *GDM*2 the value 1 indicates that for the compared objects *i, k* between corresponding observations on ordinal variables, relations "greater than" take place or relations "greater than" and relations "equal to", if they are held for other objects (i.e. objects numbered $l = 1, \ldots, n$; where $l \neq i, k$),

– it satisfies the conditions: non-negative, reflexive, and symmetric (for all $i, k = 1, \ldots, n$),

– the empirical analysis proves that distance GDM sometimes does not satisfy the triangle inequality,

– it needs at least one pair of non-identical objects in order to avoid zero in the denominator,

– the transformation of data by any strictly increasing function (*GDM*2) or by any linear function (*GDM*1) does not change the value of distance.

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